











YALE UNIVERSITY  
MRS. HEPSELY SILLIMAN  
MEMORIAL LECTURES

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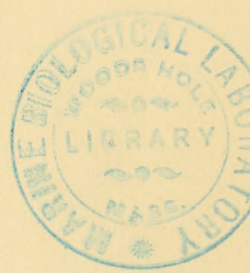
*The Anatomy of Science*



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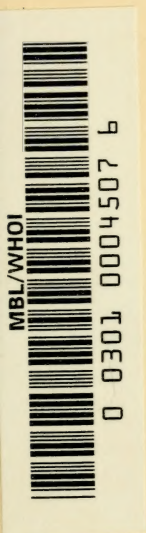
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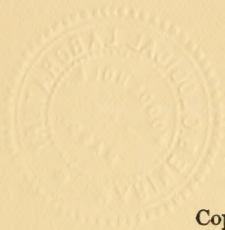


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## THE SILLIMAN FOUNDATION

IN the year 1883 a legacy of eighty thousand dollars was left to the President and Fellows of Yale College in the city of New Haven, to be held in trust, as a gift from her children, in memory of their beloved and honored mother, Mrs. Hepsa Ely Silliman.

On this foundation Yale College was requested and directed to establish an annual course of lectures designed to illustrate the presence and providence, the wisdom and goodness of God, as manifested in the natural and moral world. These were to be designated as the Mrs. Hepsa Ely Silliman Memorial Lectures. It was the belief of the testator that any orderly presentation of the facts of nature or history contributed to the end of this foundation more effectively than any attempt to emphasize the elements of doctrine or of creed; and he therefore provided that lectures on dogmatic or polemical theology should be excluded from the scope of this foundation, and that the subjects should be selected rather from the domains of natural science and history, giving special prominence to astronomy, chemistry, geology and anatomy.

It was further directed that each annual course should be made the basis of a volume to form part of a series constituting a memorial to Mrs. Silliman. The memorial fund came into the possession of the Corporation of Yale University in the year 1901; and the present work constitutes the twentieth volume published on this foundation.



## PREFACE

**I**T may be the ineradicable impulse of every living species to people the earth that has driven mankind into the present great industrial movement, whereby quantity-production is providing subsistence to an ever increasing population. Whatever the propulsive force may be, it has led us gradually and unsuspectingly into a state of socialization which in many respects has gone far beyond the demands of the professional socialist.

The slogan of this movement has been "Efficiency through specialization," and specialists we have all become. The mechanic who spends his days turning out little parts for some larger whole, that he may never have seen, has his counterpart in the scholar whose life is spent in the minute exploration of some small section of human culture, or in the repeated application of some one laboratory method. Strange though it be, even philosophy has been given over to professionals, and all about this large domain we find the signs, "No trespassing."

The conspicuous part played by science in this whole movement has been fully recognized. The public, incurious as to the spirit and methods of science, has nevertheless acclaimed its

practical accomplishments and has been eager to read of a new vaccine which rids humanity of some deadly scourge, of a new fuel or a new engine, and even of the prodigious store of energy lurking within the atom, which may some time be set free in the interest of a more stupendous industrialism.

However, while we are all obliged to keep step in this great social march, there are many of the more individualistic and the more philosophical sort who look back longingly to the days of less elaborate manufacture, when each synthesis could be watched through its various stages, and sometimes even be performed, by the amateur. To such people, who are interested not so much in the products of science as in its methods, I am offering this glimpse of the inside of the scientist's workshop,—his habits, his tools, and his raw materials.

Once as I wandered through an old graveyard in Watertown, Massachusetts, I read in the epitaph of an ancient divine that "he was a careful and painful preacher." I do not much doubt that this was true even in the modern sense, for it was once the fashion to show respect for the graver problems of life by a ponderous style and a solemn mien. Now the modes



are changed, and I shall not be suspected of levity if I treat with occasional whimsicality some of the most serious questions that arise from our study of the world of nature.

I have endeavored not to be painful and yet to be careful. Although I am chiefly addressing a popular audience, nowhere have I concealed difficulties nor have I wittingly sacrificed accuracy of thought. Indeed I am in part addressing the specialist too, and hope that he may find in the several chapters something new of substance or method.

It would be the height of presumption to claim that anything like a System of Philosophy is contained in these simple essays, which have been brought by the publishers to the dimensions of a book only by generous spacing between the lines. And yet, reading between these lines, the reader may perhaps discern the outlines of a singularly satisfying little philosophy, which I venture to believe is shared by a number of men of science, although it differs as widely from the materialism traditionally attributed to scientists as it does from the thinly disguised theology of classical metaphysics.

*Berkeley, California,  
July 14, 1926.*



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# THE ANATOMY OF SCIENCE

## I

### *Methods of Science; Numbers*

THE strength of science lies in its naïveté. Science is like life itself; if we could foresee all the obstacles that lie in our path we would not attack even the first, but would settle down to self-centered contemplation. The average scientist unequipped with the powerful lenses of philosophy, is a nearsighted creature, and cheerfully attacks each difficulty in the hope that it may prove to be the last. He is not given to minute analysis of his own methods. Indeed, if he should become too self-conscious he might lose his power, like the famous centipede who, after too profound analysis of his own method of locomotion, found he could no longer walk.

Yet, as the artist, after painstaking effort, steps back from his easel to view his picture as a whole, so it may not be unprofitable for the scientist to forsake from time to time his own specialty and survey the general trend of sci-

ence.<sup>1</sup> Mach<sup>2</sup> says, "Every philosopher has his own private science and every scientist his private philosophy," intimating that both are rather crude affairs. May we not make of these two a blend, retaining a little of the creativeness of science, even with some of its naïveté, and of the breadth of philosophy, even with some of its scepticism?

To give any sort of historical account of the development of scientific concepts would require more space than is allotted to these pages and a competence far surpassing my own. What I shall attempt to present will be a kind of contemporaneous cross section showing the inner structure of science. Such a presentation must of necessity have a somewhat personal bias. I know that I shall say many things that have been said before, often by authors unknown to me, but I shall console myself with the hope that one or two of the things that I say here will, in their turn, be said again.

I should have liked to use the word 'metaphysics' in the title of this book, but there are

<sup>1</sup> "These kinds of contemplation should therefore be alternated and taken by turns, so that the understanding may be rendered at once penetrating and comprehensive." Francis Bacon, *Novum Organum*.

<sup>2</sup> Ernst Mach, *Erkenntniss und Irrtum*.

certain words which have accumulated such evil implications that they must either be abandoned, or withdrawn for a period of purification. Two such words, 'phlogiston' and 'ether,' we shall have occasion to discuss in later lectures. However, in its best sense metaphysics might well be defined as the study of the major abstractions of the human mind, such as space, time, matter, life, love, duty, patriotism,—we need not enumerate further. A more or less complete list of our major and minor abstractions is furnished by any unabridged dictionary. There is not a word that we use which is not a product of the remarkable process of abstraction which is always associated with thought, and which perhaps *is* thought. This process of abstracting or idealizing or refining the raw material of experience is one which we shall have frequent occasion to discuss.

When we speak of the major abstractions we mean those which have been derived from a great mass of raw material through a vast number of these refining processes. The same material of which cheese is made is also used for making chess sets, but in the latter case the nature of the raw material is of little importance. The value of the chess set depends upon



the amount of human effort which has gone into its making. So we have some ideas that are freshly derived from experience, while others have passed through so elaborate and prolonged a system of refinement that we no longer know from what raw material they came. But instead of saying categorically that this idea is objective, that is subjective, may we not say merely that one idea is more subjective than another, recognizing all gradations in the extent of operation of our process of abstraction? Indeed, it will be our policy not to emphasize those classifications of ideal and real, false and true, and the like, which often give too smug a view of natural philosophy; but rather to point out from time to time the artificialities of the boundaries which these classifications set up.

When we consider those ideas which we have called the major abstractions, we find that a large group, dealing with the relations of man with man and of man with God, cannot be discussed with the same freedom from prejudice and passion as those which are less intimately human or social. The hardest study of mankind is man. Perhaps a tiger beetle or a tarantula could undertake the study with less prejudice, but perhaps also they might be distracted by



wondering whether a creature with only one third or one fourth of the harmonious number of legs could possess real intelligence.

Human beings cannot be persuaded to regard themselves as mere natural phenomena. It is an alluring thought that the true, the beautiful and the good are merely embodiments of a single idea, but it is a thought which makes little appeal to a man with an ulcerated tooth. We go on nailing up the scaffolding of our joint temple to Goodness, Beauty and Truth until a hammer hits our own thumb, and our enthusiasm for this kind of architecture rapidly wanes. If, then, by common consent we agree to segregate the natural from the social sciences it is because stars and electrons and chromosomes do not bring us so near to the quick of human feeling as religion and society and man.

The methods of natural science are not unique. If we ask how long ago the great group of Indo-European languages were only local dialects of neighboring tribes, or how long ago some fossil was embedded in a sedimentary rock, the two problems present the same kind of difficulty. In both cases we recognize that there have been periods of rapid accretion or rapid erosion, and other periods of stagnancy, but by observ-

ing rates of growth of present levels, or rates of change of living languages, an estimate can be made.

I take it that the scientific method, of which so much has been heard, is hardly more than the native method of solving problems, a little clarified from prejudice and a little cultivated by training. A detective with his murder mystery, a chemist seeking the structure of a new compound, use little of the formal and logical modes of reasoning. Through a series of intuitions, surmises, fancies, they stumble upon the right explanation, and have a knack of seizing it when it once comes within reach. I have no patience with attempts to identify science with measurement, which is but one of its tools, or with any definition of the scientist which would exclude a Darwin, a Pasteur or a Kekulé.

The scientist is a practical man and his are practical aims. He does not seek the *ultimate* but the *proximate*. He does not speak of the last analysis but rather of the next approximation. His are not those beautiful structures so delicately designed that a single flaw may cause the collapse of the whole. The scientist builds slowly and with a gross but solid kind of masonry. If dissatisfied with any of his work, even

if it be near the very foundations, he can replace that part without damage to the remainder. On the whole, he is satisfied with his work, for while science may never be wholly right it certainly is never wholly wrong; and it seems to be improving from decade to decade.

The theory that there is an ultimate truth, although very generally held by mankind, does not seem useful to science except in the sense of a horizon toward which we may proceed, rather than a point which may be reached. If there were an absolute truth of to-day, would it be the absolute truth of to-morrow? The trenchant Remy de Gourmont<sup>3</sup> writes: "There is no truth since the world is perpetually changing. You have acquired the notion of evolution . . . but you have wished at the same time to preserve the notion of truth." We have all been merry at the expense of the preacher who began his prayer, "Paradoxical as it may seem to Thee, O Lord," but our laughter is due to the very assumption that we are now questioning, that there is an absolute truth free from paradox. If we adopt the view that ideas are constantly evolving to meet an ever changing environment,

<sup>3</sup> Remy de Gourmont, *A Night in the Luxembourg*.

the hope of attaining a perfect fit between the two may be illusory.

In science a period of extreme activity in observation and experimentation is a period in which many contradictions seem to appear, old theories have to be discarded or modified, and the readjustment cannot keep pace with the new data. The physical sciences are now in such an "awkward age" of rapid growth. On the other hand, in a period in which few new observations are made, our theories have a chance of coming nearer to an adequate interpretation of our knowledge.

A paradox is never very terrifying to the scientist. Faraday wrote to Tyndall,<sup>4</sup> "The more we can enlarge the number of anomalous facts and consequences the better it will be for the subject, for they can only remain anomalies to us while we continue in error." The scientist recognizes that he is always in the midst of paradoxes and that it is his duty to resolve them. He knows that the science of the future will also have its paradoxes, but believes that every individual paradox can be resolved, that this process of resolution will lead not to greater complexity but to greater simplicity, and that

<sup>4</sup> Tyndall, *Diamagnetism*.



out of discord a more perfect harmony will evolve. This I take to be the universal *credo* of science.

Perhaps, indeed, the scientist in his attitude toward paradox is a little more reverent than those who are constantly grumbling because some flaw has been found in a preëstablished harmony. After all, is it necessary to decide as to the existence of all these ultimates and absolutes, and especially as to the existence of an absolute truth? If we once get out of the child-like notion that every act is either right or wrong, that every statement is either true or false, that every question may be answered with a 'yes' or a 'no,' we still recognize that with our present knowledge there are some statements which are more probable than others.

If we arrange various statements according to our estimate of their probability, we get the idea of a continuum, ranging from no probability to complete certainty. But whether or not any of these statements are situated exactly upon one or the other of the terminals of this continuous range may not be of great consequence. At any rate, the scientist must practice economy of thought, not only in his mode of thinking but also in deciding what to think

about. He does not find it economical to think about ultimates, and yet it must be confessed that in spite of himself he often sees them creeping insidiously into scientific thought.

Lightly as we have touched upon the problem of absolute truth, we must pass even more rapidly over the age-old problem of the real and the ideal; the objective and the subjective. If we had not studied the motion of the planets we know that someone else would, with like results; and while we turn our telescopes upon the moon, children are crying for that same moon and dogs are baying at it. On the other hand, we realize that even in the act of registering our sense impressions they are being profoundly modified owing to the instincts that have come to us from long heredity, owing to our individual memories, to our sensations, and above all, to our communications with other human beings.

How different your dog Towser would look to you if you had never seen him before, and especially if you had never seen any dog before. The word 'dog' is an abstraction from many Tow-sers, and as we cull the traits of similarity from a larger and larger mass of observations we proceed from the special to the general. Tow-



ser, dog, mammal, vertebrate, animal, living thing, object,—these are successive products of the great process of abstraction. Often as we proceed in the direction of greater abstraction and idealization, we eliminate little by little the empirical material from which the abstractions were derived, but it seems probable that the empirical is never wholly eliminated. So men gather beet roots, and, subjecting the juices to various arts of refining, obtain a sugar which still contains some traces of its earthy origin. Perhaps we know what we mean by a pure chemical, but no one has ever seen one; and I doubt if we shall ever see a pure abstraction.

It would be far from my purpose to enter upon an exhaustive study of the difficult problems in the theory of knowledge. As is remarked by Edgar Allan Poe,<sup>5</sup> himself a remarkable thinker, "The mental features discoursed of as the analytical, are, in themselves, but little capable of analysis." Nevertheless, if we are to discuss the concepts of science we must at least have a partial realization of the mode of their evolution.

I say evolution rather than manufacture in order to accentuate the point of view that I

<sup>5</sup> Edgar Allen Poe, *Murders in the Rue Morgue*.

here present. I am suggesting that the combined analysis and synthesis which we call the process of abstraction, and by which we assimilate and metabolize our observations, may best be regarded as an organic process which accompanies thought, but is not altogether subject to our own volition. We cannot make a vine grow, but we can train it in its growth. So also we can train this mental process until we call it reason, but the process itself is going on willy-nilly in the mind of a Darwin or a Jukes. Presumably we see the same process in the mind of a dog who, in recognizing the call to dinner as meaning not always meat or dog biscuit but always something to eat, is, in his turn, abstracting and generalizing.

Anyone who has once acquired the habit of regarding this process of abstraction as a living, growing thing, with creative and procreative power of its own, will never, I think, return to the idea of stagnant, man-made concepts which are but wax models of a vital and fragrant flower. Thought is a luxuriant growth, redundant and wasteful, from which here and there we may cut a branch or a twig and whittle it to our purpose.

With this view of the process of thought as

a prolific growth which we can neither start nor stop, but may sometimes direct and bend to our purpose, we no longer heed such sterile dicta as those which say that nothing comes from a process of reasoning except that which is put into it. As well might we tell the farmer that nothing comes out of the ground except what goes into it. Every thought, every statement, throws out its tentacles to search the neighboring territory. We cannot state a rule without wondering whether the converse is true. If I write down the numbers 1, 3, 5, 7, you are all expecting me next to write down 9. If I slowly draw a curve upon the blackboard, you begin to predict whether this curve will take an upward or a downward direction. In some measure our thought is always directed by our previous knowledge; and when the process of reaching out from given data is no longer a random search but is given a quite definite direction, we call the process *extrapolation*.

Now there are many kinds of thought and processes of abstraction which have little to do with the particular objective that we call science, yet, as I have already intimated, the methods are often very like those which lead to our sciences. A warlike people, during peaceful in-

terludes, amuse themselves by playing at war. Armies, with their horses and elephants, are brought together for the great manœuvres, which, however, prove to be costly things, so that wooden replicas of men and horses and elephants come to be used on a smaller ground. Then the nature of the ground and the movement of the pieces become conventionalized, and convention succeeds convention until finally we have that remarkably compact and perfect abstraction called chess.

Here is a concrete illustration of the abstracting process which may serve as an introduction to our study of the growth of scientific concepts. As we now proceed to discuss these concepts one by one, this examination of concrete cases will give us a better idea of the power and also of the limitations of the scientific method. We shall begin with the oldest of the mathematical sciences,—arithmetic, the science of numbers.

Let me start with a simple parable. A little lame boy, sitting on his doorstep, watches the schoolhouse across the way, and a class of boys at play and at work. He has heard them recite the childish formula, “Eeny, meeny, miny, mo, catch a nigger by the toe,” and has adopted

this formula for tallying objects and occurrences. When the school clock strikes after the noon hour, it says to him "eeny" and later on "eeny, meeny," and again "eeny, meeny, miny," when the class is dismissed. As the boys leave their classroom in a procession, in the order of their ages, the little lame boy checks them off with his formula, one word to a boy, and he has come to think of the oldest boy as "eeny," the next as "meeny" and so on to the youngest, who comes out last at the word "nigger."

This habit of picking out some series of familiar words and using them for tallying and naming other sequences of things or events we call assigning order-names or ordinal numbers. After the alphabet was introduced the Greeks used  $\alpha$ ,  $\beta$ ,  $\gamma$ ,  $\delta$ , as symbols to express ordinal numbers; but long before the invention of the alphabet, and even before the parent Indo-European language was scattered to the four quarters of the earth, it possessed a counting series which, with but slight changes in pronunciation, we have to-day in our one, two, three, four.

Let us return to our parable. One day the little boy had the idea of tallying the class, not as they left the schoolroom but as they entered,



in the random order of their arrival. No longer does he check off the oldest boy with "eeny" and the next oldest with "meeny," but he notices that the last boy enters at the word "nigger." The next day the boys come in another order, but again the last one enters at the word "nigger," and this proves always to be the case if the boys *are all there*. The lame boy says to himself, "If it had been a larger class the last word might always have been 'toe.'"

The boy has now made an important *observation*, and if he tries the same method with other sets of objects he discovers an *empirical law*, perhaps the most important scientific law that has ever been discovered. He finds that for any such set, regardless of the order in which the individuals occur, there is a constant something which indeed can be represented by an order-name, but this order-name now answers the question "How many?"

I do not mean to imply that historically the ordinal number came before the cardinal, nor can we know how, in course of time, the law that every set has a cardinal number was abstracted from particular sets of objects, events and ideas; nor how this law itself underwent further refinement until it now seems almost to be a

convention or mere definition. These processes of abstraction go on without our knowledge or control. So the formidable machine of mathematics is no sooner created to be our servant than, like the "Universal Robots" of a recent play, it develops a will of its own, and when once started cannot be stopped. We begin writing  $a$  for the first number,  $\beta$  for the second, and when the alphabet is done we proceed with  $aa$ ,  $\beta\beta$ , and later  $aaa$ . But having taken these timid steps we are suddenly appalled by the monster of infinity, for having set up the rules nothing can prevent our minds from hurrying on through an endless series of alphas and betas. We have no sooner begun our set of integers than it has become endless.

We might go back to the meditations of the little lame boy when a new class is added to the school and he has to distinguish between Eeny and Meeny of the big room and Eeny and Meeny of the small room, and I might relate how when the two classes were joined together he formed the notion of addition and called to his mother with the cryptic remark, "Miny and nigger make toe." But I cannot enter here upon a discussion of the fundamental operations of addition and multiplication, which have been

so ably treated in simple language by Clifford,<sup>6</sup> Poincaré<sup>7</sup> and Hobson.<sup>8</sup> Clifford's book, although published over thirty years ago, still seems to me so admirable that I have taken it, more than any other work, as my model in the present chapters.

With respect to these operations of arithmetic I do wish to say here what cannot be said too often, that as a science becomes more abstract, more removed from the empirical observations out of which it sprang, it becomes less and less profitable to inquire whether it is true. This is no splitting of hairs. Regardless of what we may think about absolute truth, no one can question the convenience of the words "truth" and "reality." Many of our ideas are so little likely to be questioned that we file them away in order to leave our desk clear for the very doubtful questions which are constantly arising, although there is no knowing when we may be obliged to take any of these ideas out of our file for further scrutiny.

A palæontologist finds a fossil bone and concludes that it belongs to some member of the

<sup>6</sup> Clifford, *The Common Sense of the Exact Sciences*.

<sup>7</sup> Poincaré, *Science and Hypothesis*.

<sup>8</sup> Hobson, *The Domain of Natural Science*.

elephant family, but to no existing species. He sketches a hypothetical creature and calls it a mammoth. Then, as more bones are found, not only the complete skeleton is deduced, but also much information regarding the position and size of muscles. Thus the mammoth is reconstructed, although there may be many disputes as to detail. Later, caves of the old stone age are found, with accurate drawings of the self-same creature. Finally, in the northern sea an iceberg is seen, enclosing the complete carcass of a mammoth thus preserved for thousands of years in cold storage for the confirmation of the work of the palæontologist and the archæologist. Somewhere in the course of these several discoveries the mammoth has changed from a speculation to a reality, and we file it away among our verities.

In every new and growing science there are many working hypotheses that never attain to any sort of reality. On the other hand, in the old and abstract sciences of mathematics, where it is hard to tell how much is mere definition or convention, the problem of reality is not so much doubtful as it is meaningless. A branch of mathematics may be interesting and satisfying to some of our æsthetic desires; it may be



useful in finding application in daily life or in other sciences; it may be ridiculous through some obvious self-contradiction, as a game would be ridiculous if some rule had been so awkwardly made as to make winning the game impossible. Or, finally, it may be simply dull, either because it has too little content or because it contains too much of complexity and detail.

I would not care to say that any of the work of the present day mathematical logicians is dull. Indeed, a part of it, as, for example, some of the recent analyses of Whitehead, is extraordinarily interesting; but I cannot help feeling that here and there in this field men are sometimes blinded by the illusion of finality. If there were within our reach absolutely true theorems provable by absolutely valid demonstrations, no effort to attain such perfection would be wasted. But if in our demonstrations we define every word with meticulous care, still someone will ask us to define the words used in these definitions.

When we abandon as chimerical the hope of immediate perfection we need not cease in our efforts to be more accurate in our thoughts, more cautious in our demonstrations. Realizing



that none of our foundations are perfectly strong, we attempt to make them all of sufficient strength and do not expend all of our effort on one corner stone. As we proceed to the upper stories of our scientific structure we may use lighter and lighter materials, but in each story we strive for uniform strength. We recognize the existence of different *levels* of logic, although in any one science we may build upon several of these levels, as will be illustrated in the next lecture.

However, this simile of a building with its several stories gives too much the idea of a man-made, lifeless thing. Let us think rather of the tree of knowledge which, while its branches are continually growing upward through the fog of the unexplored, is sending its roots deeper through the strata and substrata of our instincts and heritage, of our common thought and common speech.

The problem of language is so vast that I hardly dare to peek into it. But when the illustrious Willard Gibbs remarked, "Mathematics is a language," I cannot feel that he meant that mathematics is merely a dry assemblage of symbols (for to him mathematics was no formal thing), but rather that in some aspects lan-

guage has the properties of mathematics. As arithmetic has its numbers and operations upon numbers, so language has its words, and its operations that we call grammar. It is out of this great magma that have crystallized our logic and our mathematics.

After this long digression let us return to the question of whether we shall say that arithmetic is true or only that it is extremely interesting and useful, or, as William James would say, that it is pragmatic. It is certainly useful, and applicable to an enormous variety of problems, but is it applicable to all? Consider the simplest arithmetical proposition,  $1 + 1 = 2$ ; does one liter of alcohol added to one liter of water give two liters? No, it gives hardly more than one liter and nine tenths. If this failure of arithmetic seems trivial, let us consider the case of a large projectile which is moving away from us at the rate of 100,000 miles per second, and which in the same direction shoots out a smaller projectile, which travels from it at the rate of 100,000 miles per second. Does the latter move from us at the rate of 200,000 miles per second? We shall see in a later chapter that such a velocity is not only not attained but is unattainable.

Studying such failures of the arithmetical

method, we finally reach the conclusion that the method of counting can only be applied to things which can be counted, and we think of this as a sort of vicious circle; but it is not vicious nor is it quite a circle. It is the sort of circle that a spider makes in weaving its web. The growth of living thought is not to be repressed; and a cyclic thought is not a circle, but rather an ascending spiral which, with every turn, leads to greater heights. The vicious circle is only the shadow of such an ascending spiral cast upon the flat surface of formal logic.

As we proceed with these lectures we shall develop the thesis that all the branches of mathematics are but dummies upon which we attempt to fit our observations. To the scientist, whose life is spent in trial and error (I will not say in trials and errors), it seems a natural thing to try one kind of mathematics after another in order to find one that fits his immediate problem.

Let us now go back to our arithmetic operations. Having become familiar with addition and multiplication, we find in the course of our figuring that what we have just done must be undone. Thus having added two we may undo the operation by subtracting two; so the inverse

process of multiplication is division. We write  $2 \div 2 = 1$ ,  $4 \div 2 = 2$ , but as soon as we have done this our idiotic mathematical machine begins stamping out  $1 \div 2$ ,  $2 \div 2$ ,  $3 \div 2$ , and so on. We have suggested no operation of which  $1 \div 2$  is the inverse. The symbol is nonsense, but may it not be what Clifford calls "useful nonsense"? May we not employ in some way the antics of our absurd machine?

If we have a set of six things and cut it in the middle we get two sets of three, and it seems perfectly reasonable to express the process as  $6 \div 2 = 3$ . Therefore when we split one apple into two equal parts it occurs to us to use one of the nonsensical symbols,  $1 \div 2$ . We thus find an *application* for the new symbols, after which they are no longer ridiculous. The new idea even proves applicable to things which cannot be cut in halves. The striking of a bell or the laying of an egg are events which can hardly be divided. Yet if someone asks us the old conundrum, "If a hen and a half lay an egg and a half in a day and a half, how many eggs will seven hens lay in seven days?" we may not be able to give the answer offhand, but we know what the conundrum means. So also you know what I mean when I say that throughout the



day a clock strikes on the average six and a half strokes per hour.

Likewise the negative numbers found easy interpretation in debit and credit, and so fractions and negatives were given full standing with integers in the society of numbers. While the integers formed an infinite series, the fractions form infinite series even between two integers. Thus if we write  $1\frac{1}{2}$  between 1 and 2 and then interpolate  $1\frac{1}{4}$  and  $1\frac{3}{4}$  and then  $1\frac{1}{8}$ ,  $1\frac{3}{8}$ , etc., and this process is kept up indefinitely, we get an infinite series so *dense* that between any two of these fractions, no matter how close together, an infinite number of terms may still appear.

Yet if this process is carried on indefinitely no one of our numbers will be exactly  $1\frac{1}{3}$ . When, however, we have interpolated all of our thirds and fifths and sevenths, and so on, into our already dense and infinite series, it might be imagined that our needs would be satisfied and our instinct for abstraction appeased. But we find that we are not yet through. If we write  $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}$ , and so forth, and add these fractions together as we write them down, the total sum never will exceed 1, but will approach this number more and more closely as



we keep on increasing the number of terms. We may assume with Dedekind<sup>9</sup> that the sum of such an infinite series is itself a number, and that in this particular case the number is 1.

However, if we write the series  $\frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \dots$  we find that the sum of this infinite series converges somewhere in the neighborhood of  $\frac{2}{3}$ , but it is not exactly this number, nor indeed is it exactly any one of the numbers that we have hitherto discussed. If we include such sums of infinite series among our numbers then it is evident that we have once more generalized and extended the meaning of the word "number." For a long time, indeed, the new numbers were not formally admitted into the family, but finally were grudgingly adopted and still go by their old names "surd" (= absurd) and "irrational."

These irrational numbers may be further illustrated by means of the accompanying table. Any number which is less than 1 can be expressed by crossing out some of the terms in the infinite series  $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ , etc. The first row in the table represents that whole infinite series, the dots indicating that the table should

<sup>9</sup> Dedekind, *Was sind und was sollen die Zahlen?*; *Essays on the Theory of Numbers*.

extend indefinitely to the right. The second row represents the infinite sum  $\frac{1}{2} + \frac{1}{8} + \frac{1}{32} + \dots$ , continued according to the obvious rule of omitting every other term of the row above. The third row represents the series  $\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \dots$

Halves	Quarters	Eighths	Sixteenths	Thirty-seconds	
1	1	1	1	1	$\dots = .11111 \dots$
1	0	1	0	1	$\dots = .10101 \dots$
0	1	0	1	0	$\dots = .01010 \dots$

Now by a sort of shorthand this tabulation may be replaced by the symbols to the right which suggest our old friends the repeating fractions. In fact they are repeating fractions, except that, owing to the fact that there are ten digits on our hands, we are familiar with the decimal system of notation, while here we are using a dual system. Instead of decimal fractions these are dual fractions.

Finally, when we consider not merely such repeating fractions, but every conceivable com-

bination of 1's and 0's in such infinite frameworks as are represented in our diagram; in other words, when we have added to the integers all fractions rational and irrational, then, and only then, is our system of "real" numbers complete. The addition of the irrationals has made the number system not more cumbrous, but really more simple, and the system now has in every sense the properties of a continuum. This makes it possible to represent numerically a continuous change such as the growth of a tree with time, or the change of volume with temperature, so also it brings the number system into full correspondence with the positions of points on a geometrical line.

## II

### *Space and Geometry*

MANY ideas are associated in our minds with the word 'space.' The man on the street will say that space is what you put things into or take things out of. He is making a sort of abstraction or idealization of barrels, chests, wardrobes and the like. It would be easy to say that this is not the right concept of space, but how absurd such a statement would be. By the rules of language a word means what any large body of people suppose it to mean, and when we speak of space the idealized chests and wardrobes are surely in the background of our thought. But science demands some refinement of common ideas, and often a more limited and more technical use of language. Mathematics is even more exacting, and when we analyze the highly refined concept of space used by the mathematicians we find it to be quite similar to the concept of number. As number has its two aspects, ordinal and cardinal, answering the questions "In what order?" and "How many?" so we have two ideas of space, one involving

mere relative position and the other, measurement.

The discovery that space is akin to number we owe to Leibnitz, who also gave the best short description of the scientific notion of space when he wrote, "Space is the order of coexisting things." The mention of coexisting things implies that we have already formulated the concept of time. We are not considering this platform as it was ten years ago, this table as it is now, this manuscript as it will be ten years hence, but their relative positions now. We shall see in the next chapter how fully this view accords with modern ideas. Therefore limiting ourselves for the present to an instantaneous photograph of things, let us attempt to reduce to simple terms the idea of arrangement or spatial order.

Even the youngest children seem to possess a sense of spatial order, and indeed its rudiments may well be supposed to fall among the numerous instincts of the race. A child stringing beads sees that one bead may be trapped between two others, and this is a characteristic property of a one-dimensional space. If he is playing with blocks strewn upon the floor he finds that he can arrange them like a chain of



beads to form a wall. But now the individual blocks are not trapped; two blocks may be slid about so that they exchange places. If two ends of the wall are brought together then a block may be trapped inside the wall. It cannot escape by sliding, without breaking through the wall itself. If you lift the block over the wall the child cries, "That is not fair," for he is playing the game of two-dimensional space. Finally, if the blocks are nailed together to form a closed box, then one block left inside the box is completely trapped and neither you nor the child can find any way of getting it out without breaking the box. It is such facts that lead to the idea that space is limited to three dimensions.

Let us glance for an instant at the problem of empty space. We pour water out of a jug, but if we say that the jug is then empty it is because we have forgotten that the water has been replaced by air. But the air, in its turn, may be withdrawn by means of a pump, and though we cannot obtain a perfect vacuum we are coming nearer to it year by year. In any case we can imagine the air all withdrawn. Some years ago it would have been said that there still remains the ether, but this, as we shall see in the

next chapter, has proved to be an extremely unprofitable conception. But how is an empty space to be reconciled with the view that space is but the order of arrangement of things? If we have a number of objects in a row we do not say that the "row" has any substantial existence independent of the objects, nor need we attribute to space any objective existence. But we know so many objects with so complex a mesh of interrelationships in three-dimensional order that when a few objects are moved we try to keep intact as many as possible of the former relationships, and this we can best do by saying that space is left empty in a certain region.

In order to dwell for a moment longer upon these fundamental notions of order, which give rise to our purely qualitative ideas of one-dimensional, two-dimensional and three-dimensional space, we might speculate as to whether a conscious being could be entirely without any idea of space and yet later acquire that idea; although it must be realized that such speculation is apt to lead into the deeper quagmires of metaphysics. Let us imagine a person paralyzed and devoid of all senses save one, the sense of smell. He might have a mental life consisting of memories of a past succession of odors, with

some guesses regarding those to follow. Suppose now that the paralytic is wheeled daily over the same path through a garden with various beds of fragrant flowers. Might he not recognize an orderly succession of odors, constant day after day? If later he were wheeled, not by one path, but by many paths, through the same garden, might he not acquire, perhaps with great difficulty, some idea of two-dimensional order? For example, if the whole garden were surrounded by a circle of rosebushes, might he not recognize that he never proceeded from the unpleasant odors outside of the garden to its pleasant aromas except by passing through the fragrance of roses? Some entomologists believe that certain ants with a highly developed spatial sense depend almost entirely upon sensations of smell.

Such a possibility throws doubt on Poincaré's dictum,<sup>1</sup> "If there were no solid bodies in nature

<sup>1</sup> Poincaré, *Science and Hypothesis*. It is a remarkable evidence of the breathless speed with which we have recently been scaling some of the heights of scientific understanding that statements made less than a generation ago by a mathematician of such extraordinary acumen as Poincaré should now so frequently seem untenable. He states, for example, that geometrical space must be continuous and must be infinite, but both of these statements have since been challenged. Again he says that if through some new experiment a choice were forced between holding to Euclid-

there would be no geometry." Psychologists have given much attention to the development of spatial ideas from our various sense impressions, and I suppose that it will nowadays be admitted that all our ideas are derived eventually from the sense impressions that we and our ancestors have received. Still the process of abstraction has swept us so far from the countless individual experiences which contribute to our idea of space that it seems hardly more useful to speak of visual and tactual space than it would be to speak of visual and tactual number. But let us hasten away from this treacherous ground and introduce our discussion of geometrical measurement by a simple allegory.

Once upon a time the South Pacific Ocean was so overcast by clouds for a number of generations that the islanders had only a vague tradition of sun and stars. Nevertheless, their civilization grew. Their cocoanut plantations became so valuable that they made accurate land measurements with lines of cocoanut fiber, and these methods of measurement were codified by a great geometer, named Uli, so that

ean geometry and to the theory that light travels in a straight line, it would certainly be the latter belief that would be abandoned. We shall see in the fourth chapter how signally this prophecy has failed.

their children were taught the same kind of theorems and corollaries as our children are now taught. They ventured out but short distances in their canoes until they discovered the

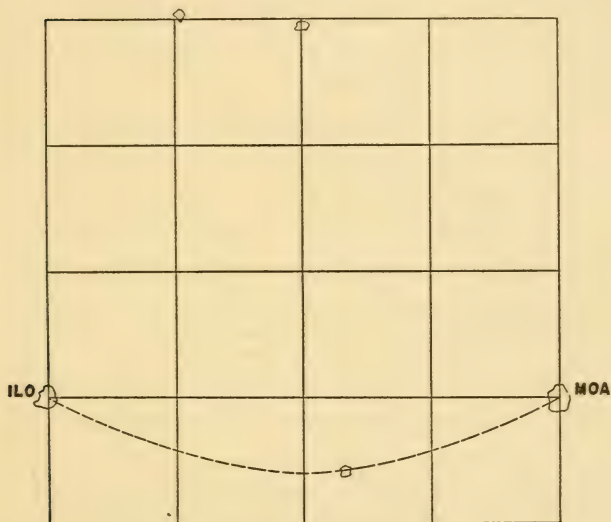


FIGURE 1

lodestone and the compass. Then their navigators became bolder and finally they had mapped out a great part of the Southern Pacific. One of their maps is shown in Figure 1. A vertical line represents the path of a canoe going di-



rectly with the compass, or as we say, north and south, and the horizontal line the path of a canoe traveling in a transverse direction, neither to the north nor to the south. The lines were placed, as they supposed, at the same distance apart, namely, ten days' paddling. They had therefore what we call a Mercator's projection. But they had no idea of a spherical earth; to them it was quite flat.

The base line of their map ran between the two islands called Ilo and Moa, forty days' paddling apart. Once a number of canoes set out on this voyage leaving Moa at the same time, each paddling by compass. But one was soon lost from the others and came upon a rocky island which the captain of the canoe knew to be far south of the regular route. He discovered that an iron amulet he was wearing had disturbed his compass, and having discarded this he set course by his compass for Ilo. Much to his surprise, he arrived there before any of the other canoes. Being of an experimental turn of mind he returned by the same route, and once more beat all his competitors. The story spread, others made similar trials, and finally they found that the quickest path between the islands was the one shown on the map by the

dotted line. It was at first supposed that ocean currents made the circuitous path the quicker one, but no such currents could be discerned. Then it was proposed that some great submerged mountain of lodestone caused variations in the compass, but all efforts to locate its whereabouts were vain.

Finally a young man, who had always been a little queer, brought forward the suggestion that the path of navigation between Moa and Ilo (the dotted line) was really a straight line, and to this he added several other statements so heretical in character that he was tried, condemned and eaten. He had no sooner been well digested, however, than other young men took up the new idea, and in particular tested his prediction that the distance between two of the northern islands shown on the map to be ten days' paddling apart would prove to be really much longer. This and other predictions being fulfilled the new advanced geometry was soon accepted and taught to mariners under the name Uliao, which means "more than Uli." It was found that the old elementary Uli was still perfectly satisfactory for the home and thence arose the saying, "Uli for the minnow, Uliao for the shark."

I must not leave my story without telling how one of the canoeists was summoned before the headman for beating his wife too noisily. He stated in extenuation that he had sat up for two hours explaining to her the principle of direct paddling between Moa and Ilo, and at the end she had said, "How very interesting, but why is it not quicker to go straight across?"


I shall not dwell upon the moral of this allegory, but I shall ask you to think of it now and then, as it seems to me to contain a good deal of the philosophy of science that I am attempting to present to you. In particular, I have tried to bring out the idea that the geometry of navigation is a two-dimensional, non-Euclidean geometry, and would very likely have been regarded as the geometry of the *plane* if it had been familiar before the earth was known to be spherical.

The geometry of Euclid has come down to us as one of the great monuments of Greek thought. For over two thousand years it has been employed almost without change as a textbook of elementary instruction. For centuries it was deemed a heresy to dispute its validity. Yet during that period it has shown its vitality by growth and change; otherwise it would

have come to us only like a fossil of ancient thought, or like some brilliant butterfly impaled in a museum case.

If we now claim a better understanding of geometry than was possessed by the Greeks, it is not through any lack of admiration of their stupendous achievement; nor is it that we believe that we are better thinkers, or that our ideas have yet reached anything like completion. I suspect that it is only the very young vine that says, "When I grow big I shall reach the sun." The older thinks only, "I climb a little higher day by day."

The geometry of Euclid is an excellent example of a structure based upon several clear-cut and distinct logical levels. In his "common notions," such as that the whole is greater than any one of its parts, Euclid presents a number of general rules of thought (or rules of language) which are useful in many other fields beside the limited field of geometry. In his postulates he states the particular rules of his geometry; and finally in his definitions he displays the subject matter,—points, lines, triangles, circles and the like. Nowadays it is the fashion to leave these elements undefined, but that is a matter of taste. No word can be com-





pletely defined, but it is sometimes useful to illustrate the meaning we desire to convey by means of examples or analogies or pictures.

It is, however, of the postulates that I wish to speak. Let me remind you of their content.

1. A straight line may be drawn from any point to any point.

2. A straight line may be produced continuously in a straight line.

3. A circle may be described with any center or distance.

4. All right angles are equal.

5. If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

If after neatly packing for a journey we should discover some necessary object and add it as an ungainly package to our carefully packed trunks and boxes, it would give the same impression that Euclid produces when after the rest of his simple and concise utterances he introduces the awkward sentence that appears in some editions as the last postulate, in others as the last of the common notions, namely, "If a



straight line falling on two straight lines, etc.”

This postulate which seems to have been and probably was inserted without much polish, at the last moment, as though it had been hoped to dispense with it, is nevertheless one of the greatest of Euclid's creations, for without such a postulate Euclidean geometry could not exist. There is nothing more interesting in the history of science than the record of the repeated attacks made on this postulate in the hope of reducing it to a proposition deducible from the other four, and which finally resulted in the discoveries by Lobachevski,<sup>2</sup> Bolyai and Riemann of other geometries, all of which are de-

<sup>2</sup> Those who are interested in the problem of the objective and subjective will find it worthy of note that the two geometries which were published independently and almost simultaneously, one by the Russian Lobachevski, and the other by the Hungarian Bolyai, were so nearly alike that they seem like different drafts of the same composition. Similarly Hamilton and Grassmann wrote at the same time those papers which were to become the foundation of modern vector analysis. We cannot avoid the thought that having embarked upon a certain line of mathematical inquiry, while we appear to have preserved the utmost of personal freedom, we seem bound to follow certain paths and to make and remake certain discoveries, just as we do in physics or chemistry. Is there then almost as much an objective world of mathematics as there is an objective world of physics? The views of a number of scientists and philosophers upon this subject will be found in the interesting book of E. Meyerson, *La Déduction Relativiste*.

ducible from the material used by Euclid, with the single elimination of this one postulate.

For our purpose we may express the postulates of Euclid in simpler form.

1. Through two points there is a unique line and this may be called the straight line.

2. About any point circles may be described.

3. Through any point outside a line one and only one parallel can be drawn.

It is the last postulate which is abandoned in the non-Euclidean geometries of which I have spoken. In the geometry of Lobachevski more than one parallel can be drawn; in that of Riemann none can be drawn. The geometry of navigation which we discussed in our allegory belongs to the last-named class. In it the straight line is what we call a great circle, and, as you know, no two great circles are parallel to each other.

The geometry of navigation would be different on a small planet and on a large one. It would depend upon what we call the curvature of the surface. And all of these non-Euclidean geometries of which I have spoken involve a certain absolute magnitude, which by analogy is called curvature. So also these are called "curved" geometries as distinguished from the

“flat” geometry of Euclid, although it is by no means implied that they find applicability only upon curved surfaces. As that absolute magnitude to which I referred becomes smaller and smaller, the geometries approach the geometry of Euclid as a limit. Moreover, the geometry of any given region approaches more nearly to the Euclidean as the region considered becomes smaller, just as in our allegory the South Sea islanders found their elementary geometry sufficient for the home.

Are these geometries true and is Euclid’s geometry false? This is a question which no longer conveys any meaning to our minds. Is chess true? Provided that a geometry contains within itself no inconsistencies or absurdities, then we regard it as true just in so far as it is interesting or useful. Certainly the laws of navigation are true and the only two-dimensional geometry that they fit is a non-Euclidean geometry. This is one of the so-called geometries of positive curvature, and all of the geometries of this class have attained enormous importance owing to recent theories of gravitation, the fringes of which we shall touch in the fourth chapter. The geometry of so-called negative curvature, typified by the original non-Euclidean geometry of

Lobachevski and Bolyai, has as yet found application only in minor ways, but its simplicity and elegance make us almost certain that it also will eventually be of great utility.

These curved geometries are already familiar to many of you, and I do not propose to discuss them further, but rather to call your attention to two geometries which have come into very recent notice, of which one bids fair to rival in importance Euclidean geometry itself. The parallel postulate of Euclid has been the storm center of geometry for centuries, but little attention has been paid to his postulate regarding circles. It is, however, by retaining the parallel postulate and abandoning the circular postulate that the two new geometries of which I am about to speak are obtained.<sup>3</sup>

First, however, let me show how in one respect modern geometrical thought has run ahead of Euclid's. The chief problem of geometry is to compare the metrical properties of one figure with those of another. Language has its words

<sup>3</sup> The most important of these two geometries (the geometry of asymptotic rotation) was clearly in the mind of Minkowski, as shown by some of the figures in his *Raum und Zeit*. The details of this geometry and of the other one (the geometry of shear rotation) were worked out in some detail by Professor E. B. Wilson and myself (*Proceedings American Academy of Arts and Sciences*, 1912).



and the rules for using words that we call grammar. Chess has its pieces and its moves, arithmetic its numbers and its operations upon numbers; geometry has its figures and its methods of comparing figures. As the rules of chess apply, not to all games, but to a single game, so the rules of Euclid apply, not to all geometries, but only to one geometry.

Now the method which Euclid used for comparing one figure with another was only a slight idealization of the method of cutting the figures out of paper and moving them about to see how nearly they fit one another. Any such method of transposing a figure may evidently be divided into two movements, one of sliding without turning, and one of turning without sliding, and we shall see that these two types of movement are closely connected respectively with the parallel postulate and the circular postulate of Euclid.

We now proceed more circumspectly than Euclid did, and we have gone further in idealizing the physical process of moving a figure cut out of wood or paper. We do not wish to be limited by the particular properties or by the imperfections of such substances, nor do we wish to be influenced by our intuition. Instead



we set up a body of rules according to which we agree to be governed as long as we are playing a particular game of geometry. Thus we make rules for what we call a *transformation* whereby a figure is reproduced in (rather than moved to) another part of our diagram, and we ordinarily make the rules such that the figure thus produced has the same intrinsic metrical properties as its original. Thus the area, the length of a certain side, the angle between two sides, we shall agree to call the same in the new figure as in the old. It will suffice to consider two kinds of transformation, one of which we may call parallel shift, and the other, rotation.

The first of these is so defined that every line produced by the parallel shift is parallel to its original. This is illustrated in Figure 2, where  $ABDC$  is the original figure, and  $A'B'D'C'$  is its reproduction. Such a transformation may be made in a single step, as shown in the figure, or in a succession or "group" of steps, each of which is itself a parallel shift.

Since in any parallel shift a line goes over into another line of equal length, it is possible not only to compare the length of any part of the line  $AB$  with the length of any other part of it, but also with the length of any part of

$A'B'$ . In other words, we may compare a length along any one line with a length along any other parallel line, but our rules so far give us no information whatever as to the relative

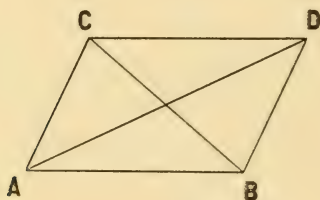
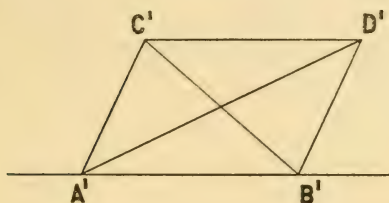


FIGURE 2

## The Parallel Shift

length of  $AB$  and  $AD$ . Certainly  $AD$  looks longer, but our eye is trained only in Euclidean methods of measurement, and we must now agree to form no such judgment until we have set up definite rules for the comparison of non-parallel lines.

In the meantime, if we had the time, we might develop the full geometry of the parallel postulate and the associated parallel transformation, and thus obtain a large number of theorems regarding triangles and parallelograms which are to be found in the geometry of Euclid, but which are also contained in the other flat geometries which I am going to describe.

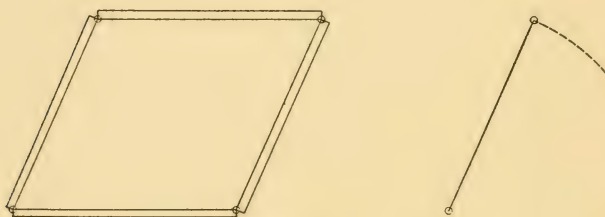


FIGURE 3

### Parallel Ruler and Compass

The theorems of which I have just spoken are only a part, however, of the geometry of Euclid; the rest depend upon the circular postulate and the associated transformation, which is called a rotation. The two transformations of Euclidean geometry may be carried out by means of the two instruments shown in Figure 3. The one on the left is the parallel ruler made of four rods hinged together; the one on the

right is the compass, which in its simplest form is a piece of string which is kept taut while one end is fixed. The first instrument we may use in all our flat geometries, but the second must be modified in accordance with the rules of trans-

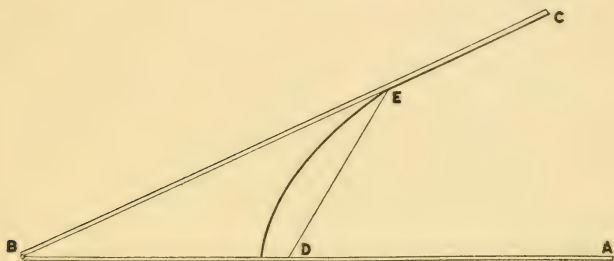


FIGURE 4

## Non-Euclidean Compass

formation which we decide to employ in place of Euclidean rotation.

Let me show you now an instrument which I may call a non-Euclidean compass (Figure 4). It consists of two rods, AB and BC, of which the first is stationary and the second is allowed to move about a hinge at B. A cord is attached to the two rods at the points C and D, and now if a piece of chalk is pressed against the string so as to keep the upper part of the string coincident with the rod BC and, maintaining this

condition, is moved slowly downward, we trace the heavy curve in the figure. If I had used a more complicated but also more complete ap-

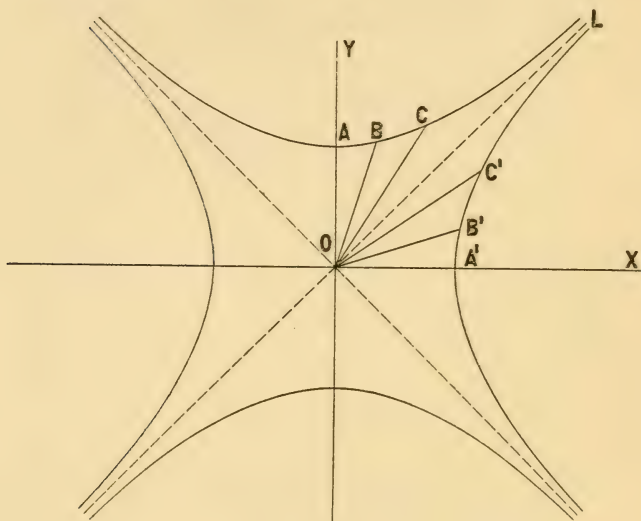


FIGURE 5

## Asymptotic Rotation

paratus, my compass would have drawn simultaneously the curves of Figure 5, which do not look like circles, but look to the Euclidean eye like hyperbolas, with dotted lines as *asymptotes* (lines which are approached indefinitely, but never reached). And yet we may define the rules



of a transformation which will make the line  $OA$  go into  $OB$  and  $OB$  into  $OC$ , while the line  $OA'$  goes into  $OB'$ , which in turn goes to  $OC'$ ; and I shall venture to call this a rotation.

Here I follow the distinguished precedent of Humpty Dumpty in *Through the Looking-Glass* who said, "When I use a word it means just what I choose it to mean—neither more nor less." Of course Humpty Dumpty and I are both wrong, for it is impossible to free a word from all its countless implications, unless we make it out of the whole cloth, as the word "gas" was once supposed to have been made. But here the point is that it seems wiser to emphasize the resemblances rather than the differences between the old and new types of rotation. You may prefer to call the new process absurd or irrational rotation, and indeed extending the word 'rotation' to cover this new asymptotic rotation is very much like extending the word 'number,' which referred originally to integers, to include fractions, and later irrational fractions.

Enlarging the meaning of rotation implies also an extension of the meaning of distance, because we have agreed to abide by the rules of our transformation in comparing the length

of two non-parallel lines. Since in our transformation  $OA$  goes into  $OB$  and  $OB$  into  $OC$ , we must say that  $A$ ,  $B$  and  $C$  are equidistant from  $O$ . Thus we lay down the rules of a geometry which is as simple and beautiful as the geometry

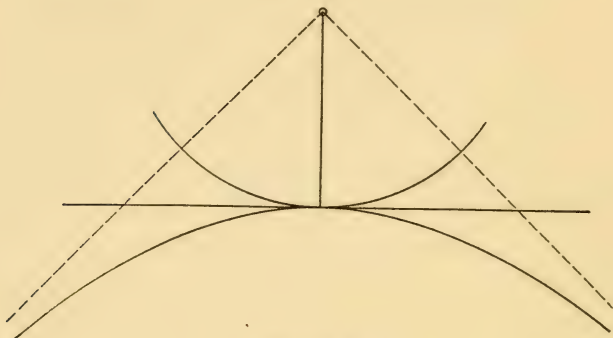


FIGURE 6

## The Three Types of Rotation

of Euclid, and which has such important applications that we may before long see this geometry taught by the side of Euclid in our schools.

Finally, we may set up rules for a third flat geometry in which the curve of rotation is neither circular nor asymptotic, but merely a straight line. The three types of curves of rotation are shown in Figure 6. The new kind of rotation corresponds to what physicists call a

shear; it is like the movement of a pack of cards when the top slides and the bottom remains stationary. The geometry of shear rotation defined by such a transformation lacks the fullness and complexity of either the Euclidean or the asymptotic varieties. It might for this reason be called a “degenerate” geometry.

Thus by taking liberties successively with two of Euclid’s postulates it is possible to obtain the four additional types of geometry shown at either side of the central Euclidean geometry in the following tabulation:

No curvature	No curvature	No curvature	Negative curvature	Positive curvature
Asymptotic rotation	Shear rotation	Circular rotation	Circular rotation	Circular rotation

The two on the right are the older non-Euclidean geometries, those on the left the newer ones. There may be other branches of mathematics which deserve to be called geometries, but these five, together with their hybrids, constitute the great family of Euclididæ.

Since I do not wish to tax your patience during these chapters by discussing matters of a technical character, this is about all that I should say regarding the non-Euclidean geome-

tries; but there are a few characteristics of the two flat geometries to which I shall find it necessary to refer in subsequent chapters. Therefore, without giving any proofs, I should like to describe a few more of the interesting features of these geometries.

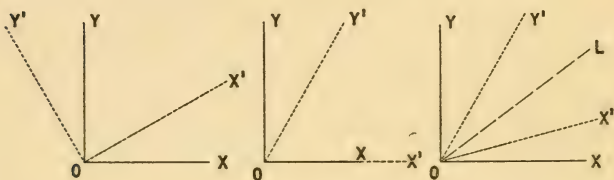


FIGURE 7

### Rotation of Perpendicular Lines in the Three Geometries

I have already shown the general characteristics of rotation in the three geometries; let us see what happens to a pair of perpendicular lines in these three kinds of rotation. In Figure 7 we see on the left the Euclidean case, in which  $OX$  and  $OY$  change into  $OX'$  and  $OY'$ . In the middle is the shear rotation, in which only one of the perpendiculars is changed in direction by the rotation, but in accordance with our rules we must still say that if  $OX$  is perpendicular to  $OY$ ,  $OX'$  is also perpendicular to  $OY'$ .

Finally at the right we see the asymptotic rotation, in which  $OX$  and  $OY$ , retaining their perpendicularity, move scissorwise toward the singular line  $OL$ , which we have called the asymptote.

The latter process is shown also in Figure 5, where the perpendicular lines  $OA$  and  $OA'$  go by rotation into  $OB$  and  $OB'$ , and these into  $OC$  and  $OC'$ , and such rotation can be repeated indefinitely while the pair of perpendiculars approach nearer and nearer to the singular line  $OL$ . It is thus impossible to rotate a line such as  $OA$  into a line such as  $OA'$ , for these two classes of lines are permanently separated by the singular lines or asymptotes (which are the broken lines of the figure). And here we may point out a very remarkable property of these singular lines. Since  $OA$  may be rotated to  $OB$  and then to  $OC$ , and so on without limit, always keeping the same length, the terminus approaches the singular line beyond any point which we can mark off upon it, and we thus reach the conclusion that any interval marked off upon one of these singular lines is of zero length.

As we proceed to develop the asymptotic geometry we are struck by the observation that almost every theorem of Euclid has its counter-



part in the new geometry, and indeed the wording of the theorems is usually identical, except for an occasional difference in sign (while theorems in the geometry of shear rotation represent a sort of compromise between the other

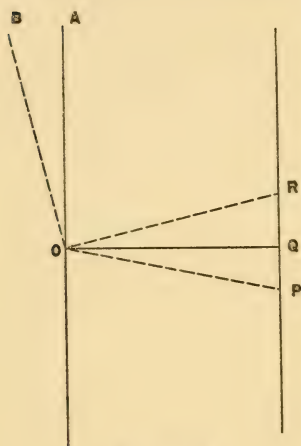


FIGURE 8

two). Thus in the asymptotic geometry the square on the hypotenuse of a right-angled triangle is equal to the difference between the squares on the other two sides.

So in Figure 8, if we have two parallel lines OA and QR, and the line OQ perpendicular to these, we may take a new line

OB and draw the perpendicular to it in the three geometries. We obtain OR in Euclidean, OP in the asymptotic, while OQ still remains perpendicular in the shear geometry. Now if  $s$  represents what we may call the slope between the lines OA and OB (in Euclidean geometry we call it the tangent of the angle between OA and OB), the ratios of the lengths of the three

new perpendiculars to the original perpendicular are for the Euclidean, shear, and asymptotic geometries, respectively,

$$\sqrt{1 + s^2}, \quad 1, \quad \sqrt{1 - s^2}.$$

Let us consider the three lines OA, OB and OC of Figure 9. If  $s_1$  is the slope between the first and second lines, and  $s_2$  the slope between the second and third, then  $s_3$ , the slope between the first and third, is given by the three following equations for the Euclidean, the shear, and the asymptotic geometries, respectively:

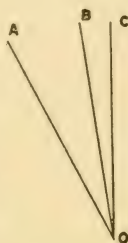


FIGURE 9

$$s_3 = \frac{s_1 + s_2}{1 - s_1 s_2}, \quad s_3 = s_1 + s_2, \quad s_3 = \frac{s_1 + s_2}{1 + s_1 s_2}$$

I shall not continue further the discussion of these interesting geometries. These few examples will serve to illustrate their general character, and these examples are the ones which we shall find useful in our later discussions.



### III

#### *Time and Motion*

WE can imagine a person without any idea of number; we have even attempted to picture the gradual acquisition of a sense of space; but it seems quite impossible to imagine a conscious being without a sense of time. Presumably our primary notion of time is due to a recognition of order or sequence in our own process of thought, or as Eddington<sup>1</sup> put it, "Our minds are immediately aware of a 'flight of time' without the intervention of external senses." This primary conception of time as a sequence of sensations and thoughts has, however, become highly complex, and in its course of development many other ideas have become interwoven with it, for example, the concept of causality. The scientist's view of time has numerous connotations which are mutually independent, and perhaps even contradictory. During the course of these brief chapters I shall be able to mention only a few of the important concepts of science, and many of these inade-

<sup>1</sup> A. S. Eddington, *The Mathematical Theory of Relativity*. See also his more popular exposition, *Space, Time and Gravitation*.

quately; but the concept of time is so very fundamental that we shall meet it in several of these chapters, and it will be one of my most interesting tasks to analyze the scientific notion of time and to attempt to resolve this complex into its components.

In order to create a science out of a great body of freely growing thought, it is necessary, first of all, to eliminate as far as possible the human and subjective elements, and such a step was taken by our remote ancestors when they decided to measure the lapse of time, not by their own feelings, but by the movements of the earth and moon. Indeed, with all our inventiveness we have yet to make a chronometer more accurate than these.

A study of the motion of bodies leads us into the science of kinematics, which is evidently something broader than geometry, since it includes all of geometry and the element of time as well. Philosophers have always included space and time as cognate abstractions, but that the partnership between them is an indissoluble one was, I think, first perceived by Leibnitz, whom we have already quoted as saying, "Space is the order of *coexisting* things," and who elsewhere said, "But space and time taken



together constitute the order of the possibilities of a whole universe." This is a remarkable prophecy whose fulfillment was heralded in the ringing words of Minkowski,<sup>2</sup> "From this time on, space in itself and time in itself shall sink to shadows, and only a kind of union of the two shall retain independence."

It has become the habit of scientists to employ the Cartesian method of diagrams whenever they wish to show graphically the dependence of one quantity upon another. Thus Figure 10 shows how the volume of a given amount of water changes with the temperature. We say that the volume is "plotted" vertically and the temperature horizontally. So, too, our automatic recording instruments give us similar diagrams; a recording thermometer shows the variation of temperature with the time.

In the same way we may record the motion of bodies by showing their positions at different times. Suppose that we have a straight string with a knot at either end, and three beads lying between these knots. We take an instantaneous photograph, shown by the lowest line in Figure

<sup>2</sup> Minkowski, *Raum und Zeit* (collected papers). The same qualitative idea was clearly set forth nearly a century ago in an almost forgotten essay by Fechner, *Der Raum hat Vier Dimensionen* (Kleine Schriften).

11, where O and P represent the knots, A, B and C the beads. At the end of a minute we lower the photographic plate one centimeter and open the shutter again, thus giving upon the same

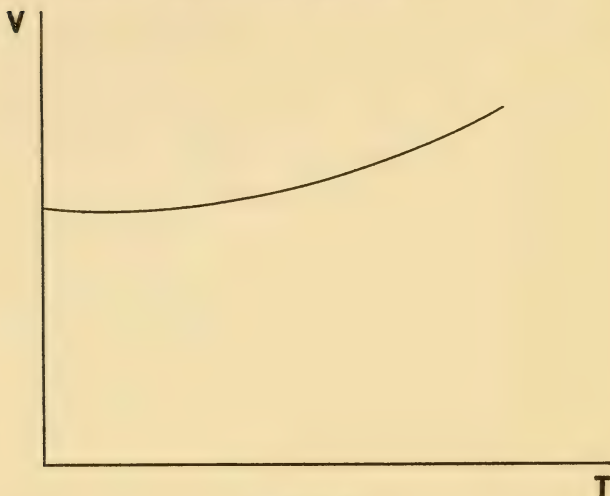


FIGURE 10

plate the picture  $O'P'$ . At the end of the second minute we repeat the process, getting the picture  $O''P''$ , and so on. Or we might have left the shutter open and moved the plate steadily at the rate of one centimeter a minute, in which case the beads would have traced continuous lines upon the plate, such as  $A A' A''$ ,  $B B' B''$ . Now when we examine the plate we see that the bead

A was stationary, the bead B was moving toward A with a constant velocity, while the bead C was moving away from A with a con-

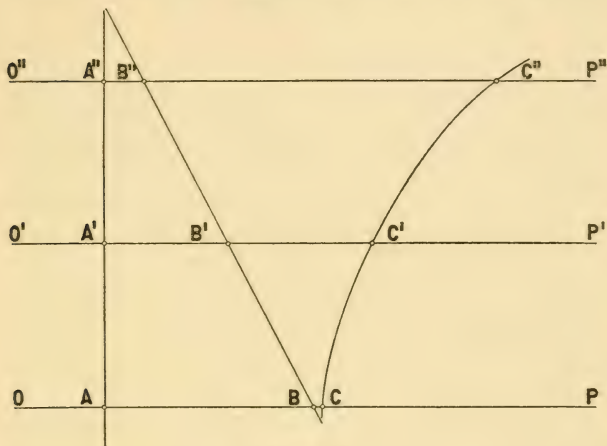


FIGURE 11

### A Space-Time Map of Beads on a String

stantly increasing, or, as we say, with an accelerated velocity. The collision of beads A and B is an *event*,<sup>3</sup> or a point in space-time, which

<sup>3</sup> It will avoid confusion in our further discussion if we agree to use the word *event* for any episode which has no considerable extension in space or time. For example, if we were dealing with the modern history of Europe the French Revolution must be treated as affecting a great many square miles of territory over a number of years; but if we were considering the whole history of the solar system, its spatial and temporal extension could be ignored and it could be regarded as an *event* in this technical sense; or, in other words, as a mere point upon our space-time map.

occurs one centimeter to the right of the knot O, and about two minutes and a half from the beginning of the experiment. The whole diagram is a *map* of the location of the beads in space and time.

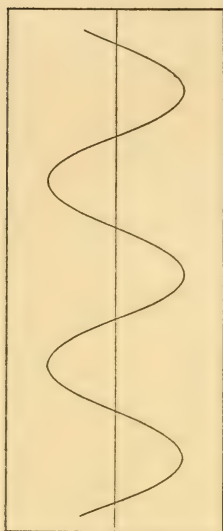


FIGURE 12

Space-Time Map of  
a Pendulum

A heavy pendulum with an ink brush attached to it traces its own space-time diagram upon a sheet of paper moving uniformly beneath it (Figure 12). Both of these figures record only motion in a single line, and this is all we can do with a plane diagram. But if we employ a solid model, using the two horizontal directions for position in space, and again the vertical directions for position

in time, we could record motions upon a plane surface, for example, balls upon a billiard table.

The sun and planets move nearly in a single plane, and I have attempted to construct a model showing the space-time map of the sun and the first three planets, a photograph of

which is shown in Figure 13. The central core represents the sun, which is assumed to be at rest, and its space-time path is therefore represented as a straight line. The innermost helix

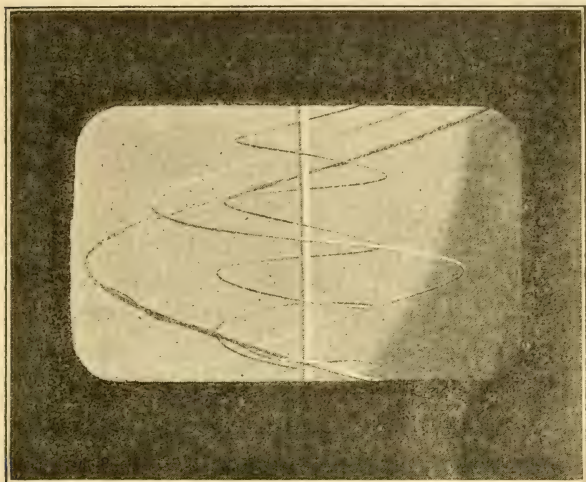


FIGURE 13

Space-Time Map of Sun and Three Planets

represents the orbital motion of the planet Mercury; the next represents Venus; and the outer one that interesting double planet, the earth and moon. This outer helix goes through one complete period of revolution, and therefore the interval of time between the bottom of the pic-



ture and the top represents a period of one year.

A complete picture of the motions of bodies in a space of three dimensions would require a four-dimensional construction, which we might represent by its projection upon a three-dimensional space, as we have represented our three-dimensional model by its projection upon a plane, but such a construction is difficult to make and also to visualize.

Now under what circumstances are we to think of a set of figures such as I have shown as constituting a geometry? I have puzzled over this perplexing question, and it seems to me that the answer is simply the one suggested in the last chapter. A geometry is more than a set of figures; it involves also the operations which permit us to compare one figure with another and thus establish rules of measurement. If we can think of any operations and transformations to which we can subject the figures that are before us, analogous to the transformations of ordinary geometry, and if these operations give interesting and useful results, then and only then may we be said to have a geometry.

I am going to apply this criterion to our pic-

torial representations of kinematics as that science was left by Newton. Without making a great effort to secure completeness or avoid redundancy, the essential postulates of Newton's kinematics may be stated about as follows:

1. There is an absolute measure of lapse of time and an absolute measure of spatial distance. If a set of (perfect) chronometers has once been brought together in a single room and set at the same hour, then however and wherever these chronometers are transported they always measure the lapse of time between two events without ambiguity. If from certain chosen bases we make a number of (perfect) surveys by measurements and triangulations, the distance between the places where two events occur will also be ascertained without ambiguity. This postulate, which we shall have to scrutinize presently, may be called the postulate of a universal or absolute time and space.

2. However, there is no unique zero hour. The lapse of time between two events is independent of the particular setting of the chronometers. Thus, it is immaterial whether the chronometers when all started together were set at twelve o'clock or six o'clock. This postulate may be called relativity of position in time.

3. Likewise the spatial distance between two events does not depend upon the particular bases from which a survey is made, or, as geometers would say, it is independent of the choice of particular axes of reference. This postulate may be called relativity of position in space.

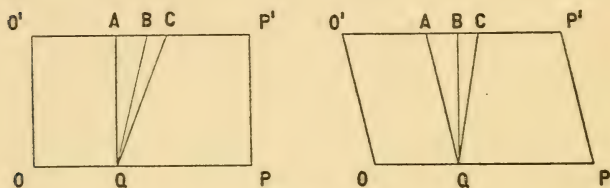


FIGURE 14

4. If two bodies are in relative motion we may equally well say that either is at rest and the other in motion. This postulate may be called relativity of motion.

It will be noted that the last three postulates deal with relatives, only the first with absolutes. From these postulates certain theorems arise. For example, if B is moving with respect to A, and C in the same direction with respect to B, then the velocity of A with respect to C is the sum of these two velocities; for in the left half of Figure 14 let us represent as before a string with three beads, at two different instants sepa-

rated by unit time (absolute). If the three beads start together and move with different velocities, QA, QB and QC would represent their space-time paths. The relative velocity of two beads is measured by the length of their separation at the end of unit time. This separation is AB for the first and second beads, BC for the second and third, and AC for the first and third; but AC is the sum of AB and BC.

So also it may be proved from the postulates that there is no limit to possible velocities, for we may have one body at rest and another body moving northward from it at the rate of one mile per second. But by the fourth postulate we can consider the second body at rest, and therefore we can have a third body moving northward from it at the rate of one mile per second, and so on without limit. Hence by the previous theorem we may find a body moving from the first as many miles per second as we please.

Let us next consider some consequences of postulates 2 and 3. If in constructing Figure 11 we had started our diagram three minutes earlier and we had reckoned our distances from an object two feet to the left of the knot O, we might have obtained a more extensive map. But the part which is shown in the figure would have

remained the same, with the same lines, and the same intervals between events, and the same slope between the lines  $AA'$  and  $BB'$  which shows the relative velocity of the two beads. In other words, the intrinsic properties of the figure are independent of a parallel shift of the axes of reference.

We see, therefore, that we are very close to having a geometry. Let us see whether the postulates also imply any operation analogous to rotation. Once more looking at the left side of Figure 14, we see the string with knots  $O$  and  $P$  represented as stationary, as is also the bead  $A$ , while the beads  $B$  and  $C$  are moving. But by the postulate of the relativity of motion we might take the bead  $B$  as at rest and the string and other beads as in motion, so that the right-hand diagram of Figure 14 would equally well represent the facts. But this suggests at once the non-Euclidean geometry which we discussed in the last lecture and which is characterized by shear rotation.

Indeed, as we pursue the inquiry we find complete identity, detail by detail, between the kinematics of Newton and this peculiar geometry in which now any distance along the parallel lines  $OP$ ,  $O'P'$ , represents a measurement with



a yardstick, distance along any other line represents readings of a clock, while the relative slope between two lines such as QA and QB represents a relative velocity. Thus we see that the theorem in the shear geometry which we mentioned in the last chapter, namely, that the slope between lines QA and QC is equal to the sum of the slopes QA to QB and QB to QC, or  $s_3 = s_1 + s_2$ , is identical with the law of the addition of velocities which is characteristic of the Newtonian kinematics.

If it had been known earlier that the kinematics of Newton could be reduced to a geometry, indeed to one of the simplest of all geometries, might not some Minkowski of an earlier day have used with equal justice the words that I quoted at the beginning of this chapter? In any case, the discovery of this coincidence would have increased confidence in the correctness of the kinematics and aroused interest in the study of that geometry; for when a certain kind of mathematics fits a large group of experimental facts we feel sure that it will also agree with the next fact to be discovered. We say that nature would not play us so scurvy a trick as to make our mathematics fit at so many points and fail

at the next one. *This I take to be the whole significance of a mathematical proof in physics.*

However, if we find that our geometry fails even in the slightest degree to coincide with our observations in kinematics, we must beware, for it will surely fail again; and when it does we must not say that our mathematics is wrong, but only that we have chosen the wrong mathematics! Supposing that in some plot of experimental data we have a number of points representing different measurements. We have on our desk a box of circles, ellipses, hyperbolas and parabolas cut out of cardboard, and by trial we find that a parabola fits the points. We then run a pencil along its edge and by this extrapolation we hope to predict a new observation; but performing our experiments we find a point which does not at all fit the curve we have drawn. We do not throw our cardboard parabola into the waste basket and say it is no good; we put it back into the box to be used on another occasion and select a new curve, perhaps a hyperbola, which now fits all the old points and also the new.

Was the simple mathematics of Newtonian kinematics going to continue to be adequate with the advance of experimental physics? The

first discovery which might have given rise to doubt was due to the Danish astronomer, Römer (1675), who noted that the frequency of revolution of the satellites of Jupiter appeared to be slightly greater when the earth moved toward Jupiter than when it moved in the other direction; just as the whistle of a locomotive seems to have a higher pitch when it is coming toward us than when it is receding. Römer rightly explained this by assuming that light moves with a finite velocity. This discovery at once reopens the question of simultaneity. We can no longer say that two events which we see at the same time are simultaneous. A bright nova suddenly appears in the skies, but we say that it really flashed out thousands of years ago and that the light has been traveling to us ever since. If we knew the distance of the star and the velocity of light we might make due allowance for this time of travel, but is the velocity of light dependent perhaps upon the motion of the earth, or upon the motion of the star that we are observing?

This was a question which later was to give much concern to physicists. But in the meantime the velocity of light came to be regarded as one of the fundamental constants of nature,—more fundamental, indeed, than light itself; for

this same constant, 300,000 kilometers per second, was discovered by Weber in his study of electrical charges and magnets. If we had never known light or any other form of radiant energy this constant would still play an important part in all of mathematical physics.

Of course the numerical value of this constant depends upon whether we measure velocity in kilometers per second or miles per minute, and it would be a great simplification of many of our equations of physics if units were so chosen as to make this velocity itself equal to unity. This suggestion has been repeatedly made throughout a half-century,<sup>4</sup> and will certainly be generally adopted before long. It will simplify my task if I adopt it here. Thus if we keep the centimeter as the unit of length, we must choose a very small unit of time, which I have somewhat jocularly called the jiffy,—so that the velocity of light becomes one centimeter per jiffy.

The repeated appearance of a unique velocity in the various branches of physics is in no way suggested by that geometry which we have shown to be a counterpart of the Newtonian kinematics, and therefore at once arouses suspi-

<sup>4</sup> See Sundell, *Philosophical Magazine*, 1882.



cion that we have chosen for kinematics the wrong kind of mathematics. But the verification of the suspicion was greatly delayed by a certain obsession which had taken a strong hold upon the minds of the physicists. I refer to the luminiferous ether. Often in our more carefully cultivated gardens of thought some rank weed grows with such vigor as to stunt the growth of the neighboring useful vegetables. So the scientific literature of the nineteenth century was overgrown with a discussion of the ether, its stresses and its strains, its density, its movement with the earth or through the earth. A mechanism that we designed to be a servant had become our master; until now that we are suddenly freed from this obsession we feel as if awakened from a hideous nightmare.

Let us examine for a moment this ether theory. Imagine two hostile gunboats anchored in a tidal river. One of them is being shelled by the other, but little damage is done because every time a shell is fired the discharge of the gun produces a wave in the river which, arriving at the other gunboat before the projectile, gives the crew warning to drop behind shelter. But after a time the crew is greatly discon-





certed to find that the shells are arriving ahead of the water wave. The captain explains that this is due to the fact that the tide has changed and that the water waves, which before were running with the tide, are now running against it.

Now in the early discussion of light its propagation seemed more to resemble the propagation of waves upon water than the course of a projectile through the air, and therefore it was assumed that light must have a carrier, and this carrier was called the ether. If a searchlight were playing upon some object, the way in which light travels would thus be supposed to depend, not only upon the searchlight and perhaps upon the object and the kind of light, but also upon the properties and motion of the ether which carried the light. This assumption has proved to be entirely gratuitous, serving to complicate a simple problem. Yet this belief in the ether, together with the belief in the validity of the Newtonian kinematics, held despite the numerous presages of paradox, which became the more alarming as experimental physics began to deal with velocities, not of a few feet or a few miles per second, but with objects like

the particles sent out from radium, which move with a velocity near to that of light.

The paradox suddenly appeared in the celebrated investigation of Michelson and Morley. This set the mathematical physicists violently at work, and it was pointed out by Fitzgerald and by Lorentz that the paradox could be at least temporarily avoided by assuming that all material objects in motion are shortened in the direction of their motion. The great work of Lorentz, which was a masterly effort to express all new data in terms of the existing theory, really paved the way for the new theory which was to replace it. He was obliged to assume that, in addition to the universal time of Newton, each moving system has a separate time system of its own, but it was Einstein who saw that all of the enormous complexities which had developed in kinematics could be swept away by regarding any one of these local time systems as just as valid as any other, and thus doing away with universal time. So far-reaching were the results of this simple idea that the same stroke of the eraser which wiped out universal time also wiped out universal ether. Ether is left as a beautiful word which may again find

a use in science, but only after it is purged of all its misleading implications.<sup>5</sup>

Einstein's theory of relativity can be expressed in many ways. Perhaps the simplest is the statement that, of all the possible velocities with which matter or energy can travel, the velocity of light is the maximum. In other words, if we reckon velocities in centimeters per jiffy, all velocities lie between zero and one. Now there are some who have been irritated by this statement, and especially by the statement that no velocity greater than the velocity of light can be properly conceived. It seemed to them, as the Volstead act seems to others, to be an arbitrary

<sup>5</sup> Historically the Michelson-Morley experiment was directly responsible for the discovery of relativity, but, as so often happens in science, it was almost immediately seen that there were plenty of other data, some of which had been in the possession of physicists for over half a century, from which the same principle could have been deduced. Now the experiments of Michelson and Morley are being repeated with a care even exceeding their own, and certain effects are being observed, small compared with the ones Michelson and Morley sought in vain, but nevertheless of great consequence to physics and astronomy if they can be thoroughly corroborated. But whatever interpretation may be placed upon these results, it seems hardly conceivable that the principle of relativity could be taken from us, although our ideas of kinematics might be extended; nor is it at all likely that physicists will be obliged to adopt once more the luminiferous ether.

limitation of individual freedom. But let these people try to conceive of a temperature below  $-273^{\circ}\text{C.}$ , let them think of an angle whose sine does not lie below zero and one.

We have seen that the first postulate of Newtonian kinematics required that there be no limit to possible velocities. Einstein's discovery therefore means that we must abandon the postulate of a universal time and the geometry that went with it. We thus arrive at what may be called a non-Newtonian kinematics, and when we look for a geometry to express the new kinematics we find one which proves to be adequate even in the minutest detail. It is the remaining one of the flat non-Euclidean geometries; the geometry of asymptotic rotation that we owe to the genius of Minkowski.

If we remind ourselves of this geometry by means of Figure 5 of the last chapter, we see that the singular (broken) lines divide all the lines through O into two classes, represented by OA' and OA. And now we correlate this geometry with kinematics by stating that any distance along a line such as OA' corresponds to readings with a measuring rod, while any distance along a line such as OA corresponds to read-



ings of a clock. The statement that OA, OB and OC are of equal length is equivalent to the statement that if the whole figure represents an ordinary space-time map, and if OA, OB, OC are the space-time paths of three clocks, all of which struck eight at the point O, then if the first strikes nine at A, the second will strike nine at B, and the third at C.

Let us pause for a moment to consider the effect upon our concept of time of identifying kinematics with a geometry. In a geometry it would be ridiculous to find a certain theorem true regarding a geometrical figure, but false if the figure is turned upside down. There can be no such dissymmetry between up and down. So in a pure kinematics there can be no dissymmetry between past and future. Indeed, if we consider the solar system and the events in space and time that we call eclipses, we know if astronomical measurements are made over a period of years we can calculate the eclipses of one thousand years hence; but we can also calculate the eclipses of one thousand years ago, and with precisely the same accuracy. On the other hand, in daily life we recognize a remarkable dissymmetry between past and future; wit-



ness the collapse of a building, the burning of a forest. How ludicrous is a moving picture film which is reeled backward! This dissymmetry enters, not in the pure science of kinematics, but in some further and more complex stage of science, and we must be on the lookout for its first appearance. Finally let me express my opinion that the phrase which is commonly heard to-day, "Time is the fourth dimension of space," is highly misleading. Neither should we say that space is the second, third and fourth dimensions of time. What we mean to say is that the three dimensions of space and the one dimension of time may be combined, and naturally do combine, to form the simple four-dimensional manifold of kinematics.

It would be out of the question to enter here upon an exhaustive demonstration of the identity between the kinematics of relativity and the geometry of asymptotic rotation, but I may give a few examples. One of the most striking of Einstein's results is the following. If three bodies are moving in the same straight line, and if  $v_1$  is the relative velocity of A and B, and  $v_2$  the relative velocity of B and C, then  $v_3$ , which is the relative velocity of A and C, will not be

the sum of the other two but will be given by the equation

$$v_3 = \frac{v_1 + v_2}{1 + v_1 v_2}.$$

Thus if each of the two velocities is nearly the velocity of light, or nearly unity, the sum will remain less than the velocity of light. This equation merely *restates* the law of the addition of slopes that we have found for this non-Euclidean geometry.

Fizeau, in the middle of the last century, performed a very interesting experiment in this connection. Light passing through a medium like glass or water travels with less than *the* velocity of light (the velocity in free space). Now supposing that we have light passing through a moving stream of water, will the velocity be the same as if the water were at rest, or will it be that velocity plus the velocity of the water? The experiment was tried and gave a result lying between the two, and this was believed to show a partial dragging of the ether with the stream of water. Many pages of mathematical physics were devoted to the discussion of why the ether happened to move in just this

way. As a matter of fact, the velocity observed by Fizeau is simply that which is calculated from the equation we have just set down.

Next let us consider that mysterious shortening of a moving object which was assumed by Fitzgerald and Lorentz and attributed to some mechanical force. But by the principle of relativity we see that this shortening can be reproduced either by setting a body in motion or even by thinking that it is in motion. Now a meterstick will trace in time and space a sort of ribbon. If we look back at Figure 8 of the last chapter, the two parallel lines OA and QR may represent the paths of the two ends of the stick, and if we assume it to be at rest we draw our time axis along one edge and our space axis perpendicular to this, OQ, so that all the points on the line OQ will represent positions of the various particles of the meterstick at a given instant. The distance OQ now represents the length of the meterstick. Next let us assume that the meterstick is in motion, and therefore draw another time axis OB, the perpendicular to which is OP. And now the length OP of the meterstick has diminished, as we saw in the last chapter, by the ratio  $\sqrt{1 - s^2}$ , which is the same as  $\sqrt{1 - v^2}$ . This is precisely the well-known

equation for the shortening of moving objects. Another of the most important and certainly the most startling consequences of the principle of relativity will be mentioned in the fifth chapter.

All of the illustrations that I have chosen are such as can be represented in plane diagrams, but a full appreciation of the wonderful parallelism between the new kinematics and the new geometry can only be obtained after examining the complete four-dimensional manifold of space and time. There it is possible to build up a body of geometrical propositions which prove to be identical in every detail with the complex theorems of the science of electricity and magnetism.

I am afraid that I have made but a dull business of this explanation. The principle of relativity is now an old story, but I wish that I could have imparted to you a little of that thrill of excitement which came to us when we first heard the strange but alluring doctrine. I remember one long summer night through which Richard Tolman and I raised one objection after another, but always in vain, until we were convinced of the truth of relativity, and when with this conviction I, a young chemist, had the

temerity to present the first paper on the principle of relativity before the American Physical Society it was with a sense of exaltation like that which one experiences on hearing for the first time a great symphony or a great poem.

Then felt I like some watcher of the skies  
When a new planet swims into his ken;  
Or like stout Cortez—when with eagle eyes  
He stared at the Pacific, and all his men  
Looked at each other with a wild surmise—  
Silent, upon a peak in Darien.





## IV

### *Matter in Motion*

FROM geometry we proceeded to the larger science of kinematics, which includes geometrical measurements and measurements of time as well. Just as we employ geometry in the practice of mensuration without regard to the composition or the intrinsic properties of the objects measured, so in kinematics we are dealing with the laws of abstract motion without caring what it is that moves. If we now wish to enlarge this science still further, so as to include not only space and time, but some other measured quantity, we seek among the countless properties of natural objects some one property which they all possess and which seems suitable for the purpose. The property so chosen by the pioneers in science is the one we now call mass, and the general science which results is called mechanics.

The choice was by no means compulsory. Electric charge, for example, is a very important property, and the great science of electromagnetics can be built up out of the concept of space and time and electric charge without

recourse to the concept of mass or to the laws of mechanics. The two sciences do, however, overlap here and there, and their interrelationships have suggested certain mechanical theories of electricity and magnetism which, if the order of development of the two sciences had been changed, would more frequently have appeared as electromagnetic theories of mechanics.

In proceeding from geometry through kinematics to either of these larger sciences we are getting farther away both from the formality of mathematics and from everyday experience; we depend more upon evidence furnished by new observations and by experiments with more or less elaborate apparatus. Yet the transition is a gradual one, and the remarkable developments in mechanics and electromagnetics which are now going on promise to reduce these two sciences to a more comprehensive but also a more difficult geometry.

In introducing the science of mechanics, we may dwell for a moment upon the earlier development of the principles of statics, which is so admirably set before us in the beautiful treatise of Mach.<sup>1</sup> When Archimedes presented his dis-

<sup>1</sup> Mach, *The Science of Mechanics*.

coveries regarding the lever he began with what seemed to him the self-evident assumption that "magnitudes of equal weight acting at equal distances are in equilibrium." When we have, as in Figure 15, two precisely similar bodies

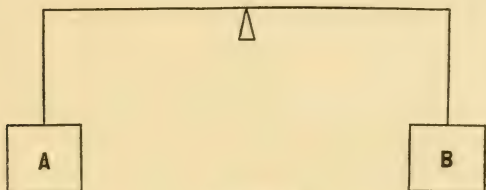


FIGURE 15

Illustrating a Theorem of Archimedes

suspended at equal distances from their mutual point of support, it would have surprised Archimedes to learn that these might not be in equilibrium, and that certain inequalities in the earth's crust might cause one body to sink and the other to rise. But if there is no dissymmetry of external conditions it becomes evident that there must be a state of balance, from what is called the principle of sufficient reason, or in mathematical parlance, the principle of symmetry. Suppose, however, that the two bodies are not just alike, that they differ in chemical substance or in shape or in color. What *a priori*

evidence have we that it is none of these properties, but the property which we call weight or mass that determines equilibrium?

It seems rather that we must define weight as that characteristic of bodies which determines balance under such circumstances. But are we justified in making such a definition? Does it not involve the experimental observation that if A balances B and B balances C, then A balances C; or, in other words, that two weights equal to the same weight are equal to each other? Often in science we find empirical laws concealed in what purports to be a definition.

If we should proceed to analyze Archimedes' various deductions we should find none of his proofs entirely satisfactory; but is there any such thing as absolutely rigorous proof? Is not a proof only an attempt to render plausible new statements by correlating them with others that are already accepted? We may doubt whether a scientist ever presents all of the evidence which has led him to a new law or a new theory. He probably is never aware of all his reasons, or at least never aware of them all at any one instant of time. Moreover, he often suppresses some part which cannot be presented with that formality which the fashion of the



day dictates, and sometimes he may even introduce irrelevant arguments for a similar reason.

Can we believe that Jean Rey,<sup>2</sup> who first announced clearly the law of conservation of mass, was himself convinced by the argument that he offered to the public?: "Let there be taken a portion of earth which shall have in it the smallest possible weight, beyond which no weight can subsist: let this earth be converted into water by the means known and practised by nature; it is evident that this water will have weight, since all water must have it, and this weight will either be greater than that of the earth, or less than it, or else equal to it. My

<sup>2</sup> Jean Rey, *The Increase in Weight of Tin and Lead on Calcination* (1630). Alembic Club Reprints. There are so many gibes at the lack of modesty among scientific men to-day that I cannot resist quoting the conclusion of Rey's extremely important paper.

"Behold now this truth, whose brilliance strikes the eye, which I have drawn from the deepest dungeons of obscurity. This it is to which the path has been hitherto inaccessible. This it is which has distressed with toil so many learned men, who, wishing to know it, have striven to clear the difficulties which held it encircled. Cardan, Scaliger, Fachsius, Cæsalpinus, Libavius, have curiously sought it, but never perceived it. Others may be on its quest, but vainly if they fail to follow the road which I first of all have made clear and royal: all others being but thorny footpaths and inextricable byways which lead never to the goal. The labour has been mine; may the profit be to the reader, and to God alone the glory."

opponents will not say that it is greater, for they profess the contrary, and I also am of their opinion: smaller it cannot be, since we took the smallest weight that can exist: there remains then only the case that the two are equal, which I undertook to prove." This learned physician was furnishing the sort of proof which was in vogue in his day. If we accept his conclusions without accepting his demonstration, it would be a precedent which we may often follow profitably with respect to the many demonstrations which follow the fashion of our own age.

Let us proceed to another stage in the sophistication of mechanics, and look at the extremely ingenious demonstration of the properties of the inclined plane as given by Stevinus in 1605. Figure 16 reproduces the frontispiece of his book and illustrates his theorem that if two inclined planes have a common base, and one plane is twice as long as the other, one body on the shorter plane will pull as strongly as two such bodies on the longer plane. The chain of balls represented in the figure will not, he says, start to go round, for, since conditions entirely similar to the original one would be constantly repeated, if it should move at all it would move

for ever. The whole is therefore in equilibrium, but since the lower loop of the chain is symmetrical it could be cut off without destroying

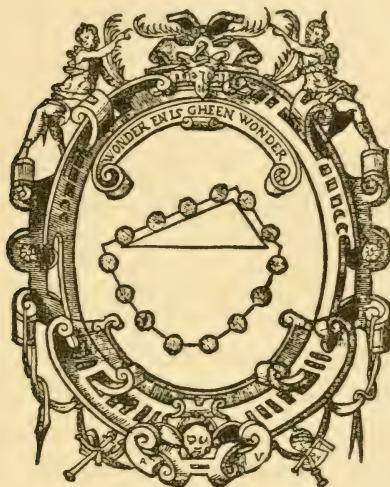


FIGURE 16

The Frontispiece of Stevinus  
(From Mach's *Science of Mechanics*)

the equilibrium, and we should have left the two balls on the one plane balancing the four on the other.

You will say that Stevinus is assuming the impossibility of perpetual motion, and that this is only another way of stating the same prin-

ciple of the inclined plane. Yes, but how would we know that there is any connection between the pull exerted by a body on an inclined plane and the idea of perpetual motion, were it not for the work of men like Stevinus? Such deductions of the unknown from the apparently irrelevant known seem to be the very gist of the scientific method. The motto which he places above his diagram, "A wonder, and yet no wonder," is an epigrammatic statement of this same thought.

It was not long after the appearance of the work that I have just mentioned that the greater science of dynamics was born in the mind of Galileo. We who step daily upon the accelerator, and note by our speedometers the immediate change in our velocity, find it hard to realize that this notion was so new in the time of Galileo that he found it necessary to coin the word "acceleration." His discovery that, except for the resistance of the air, a body falls with a constantly increasing velocity, or, in other words, with a constant acceleration, and that any two bodies, regardless of difference in weight or in other properties, when started together always keep together in falling, marked the beginning of the new science.

This law of Galileo may be readily pictured in our space-time map. Let A (Figure 17) be a spot on the surface of the earth, and B and C two bodies immediately above it. (Since we have allowed extension in time to usurp the vertical direction, we shall have to twist our heads a bit in order to visualize B and C directly over A.) The spot A traces the time-space path  $AA'$ , and the two bodies the paths  $BB'$

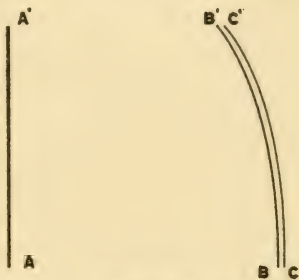


FIGURE 17

and  $CC'$ . The law of Galileo now states that no matter what bodies are represented by B and C, their space-time paths, if they start together will remain together throughout. The curvature of these paths represents the acceleration of falling bodies.

Galileo did not associate this phenomenon of falling bodies with the motion of the planets, nor did Kepler, whose stupendous researches led to as complete a knowledge of orbital motion as anyone possessed before the present century began, and who attributed planetary motion as well as terrestrial tides to an attracting force.



It was Newton, as you know, who welded together the results of these two investigators into a great law which for over two centuries stood as one of the really exact laws of nature. Indeed, the whole of mechanics as left by Newton, while it gained something of generalization and of elegance in presentation, remained without essential change until the recent remarkable discoveries of Einstein.

I shall not dwell upon the laws of classical mechanics which are familiar to all of you, but shall proceed at once to show how these fundamental laws may be displayed with remarkable simplicity in our four-dimensional geometry of space and time, or, for the sake of easier visualization, in the three-dimensional geometry characteristic of the motion of balls upon a billiard table, and the two-dimensional geometry which shows the motion of beads upon a string.

Let us represent in Figure 18 the space-time paths of two bodies which, when they are near together, exercise an action (in this particular case a repulsive action) upon one another. The masses of the two particles we may represent by the length of arrows drawn along the paths. Thus the mass to the right is twice as great as that on the left. Newton's first law of motion,

that a body in the absence of other bodies moves with a uniform velocity, may now be replaced by the law that every path is a straight line

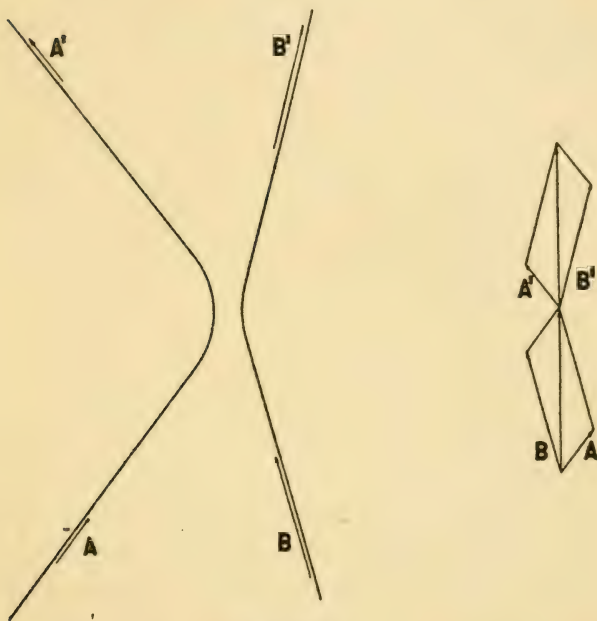


FIGURE 18

Space-Time Map of Two Mutually Repellent Bodies

except in the neighborhood of other paths. So in the diagram the two paths approach straight lines as they are farther removed from one another. As the bodies approach one another their

mutual velocity diminishes, or, in other words, as the two paths approach one another they experience a curvature.

Now, from a knowledge of the direction of the paths at the bottom of the figure, can we make any prediction as to their directions at the top of the figure? In one corner of the diagram let us draw an arrow equal and parallel to the arrow A, and another from the same point parallel and equal to the arrow B, and complete the parallelogram, at the upper corner of which we now draw an arrow equal to A' and another equal to B', and complete this parallelogram. Now we may announce the extremely simple law that the diagonals of the two parallelograms so drawn must lie in the same line and be equal to one another. This law, which Wilson and I first stated and called the law of conservation of extended momentum, comprises Newton's law of the conservation of momentum. Expressed in another way, the lower diagonal represents the locus of the center of gravity of the two bodies before they come together, and the upper diagonal represents the locus of the center of gravity after they have separated. That these two diagonals are in the same line means that the center of gravity proceeds with a constant

velocity, and that the two diagonals have the same length means that the sum of the masses is constant.

If I had the time, and you the patience, I might show you more fully how simple is the description of mechanics in terms of the new geometry; how fundamental are the little arrows that I have drawn in Figure 18, and how shadowlike are those projections (upon arbitrarily chosen space and time) which we call velocity, momentum, mass and energy.

If in my very hasty survey of the science of mechanics I say nothing of the concept of force, it is because this concept has fallen a little into the background in modern physics, and not because I share to any great extent the feeling that force is a bad physical concept because it emphasizes cause and effect. Force can be defined so as to have no causal connotation, and, on the other hand, one who is looking for cause and effect can always find it. Thus, such a one would say, with reference to Figure 18, that the neighborhood of one locus *causes* the curvature of the other.

It is indeed a serious question whether the concept of causality can be regarded as a scientific concept, and therefore within the scope of

this work. In childhood's simple vocabulary the chief word is "Why?" And as man approaches the other grand terminal of life's adventure his heart surges with the same wonder and the same question. This "why?" is the instigation of scientific inquiry, but is it the proper subject matter of such inquiry? If the idea of cause enters as a necessary and essential part of descriptions in physics, then it is a physical concept, but if it is an alien concept, serving only to blur the accuracy of these descriptions, then it lies beyond the purpose of our present discussion.

Having two rectangular pieces of cardboard with the two right-hand curves shown in Figure 19, I draw in the two curves at the left, and being interested in the problem of causality I ask a friend to account for the new curves. He, being a matter-of-fact person, says, "They are there because you drew them," and this is perhaps the best answer I am going to receive. But I try again with an artist, who answers: "These lines are drawn in accord with simple principles of design. The first case is the more elementary as the one curve is balanced by a completely symmetrical curve. But when one of the curves is heavy it must, in order to produce æsthetic bal-



ance, be nearer the center and less curved, as you have drawn it." I then go to a mathematician, who says: "In the upper diagram the

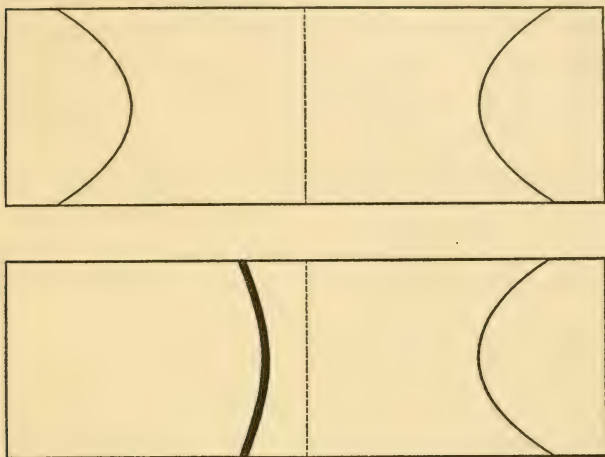


FIGURE 19

right-hand curve is a hyperbola, and if you obtain the algebraic equation which represents the position of its several points, that same equation will give the points of the left-hand curve which you have drawn. One branch of the hyperbola suggests, and you might say causes, the other branch; either of them is by itself as incomplete as a semicircle."

Finally I approach a physicist, who says:

"You are probably drawing in space and time the paths of two mutually repellent particles. In the first case the two have the same mass and each produces upon the other the same acceleration. In the second case the left-hand mass is four times as great, and therefore its acceleration is one fourth as small. The dotted line in the center probably represents the path of the center of mass." "But," I say, "why does the smaller mass have the greater acceleration?" He answers, "Suppose that the large mass is really a group of four small ones, each identical with the body on the right; if each of these four causes the right-hand body to accelerate as much as if the other three were absent, then the result will be as shown." Other questions occur to me, but I forbear.

Our search for cause leads to such a varied assortment of replies that I feel causality to be a concept which has not yet been tooled and polished to the nicety required by modern science. This view of the scientific inadequacy of the idea of cause and effect will certainly be approved by anyone, if such a one there be, who will subscribe to the extremely heterodox conclusions that I shall venture upon in the next chapter.

While we are indulging in this philosophic discussion, I may point out another set of ideas of which we are reminded by the geometrical treatment of mechanics. I refer to the problem of individuality and identity. When a caterpillar emerges from the egg, later to hide himself in his cocoon, and again to appear as a butterfly, what is there throughout these kaleidoscopic metamorphoses that justifies us in speaking of the continuous existence of a single individual? I shall not hazard an answer to this question, but only point out that an entirely similar problem is presented in physics. Our space-time map represents a material object as a sort of endless filament with an infinite extension in time, but not in space, and the same thing is true if we consider our protons and electrons which are at the present time believed to maintain their individuality through the ages. It is perhaps not quite true to state that an electron is assumed to occupy a sharply circumscribed region of space, for we ascribe to the electron a field which extends to infinite distance, and we sometimes think of the electron as nothing but this field. Nevertheless, this extension in space is quite different from the assumed extension in time, and does not prevent

the space-time map of a material system from presenting a highly fibrous or stringy appearance. Will this unsymmetrical extension with respect to space and time ultimately be regarded as a flaw in that space-time geometry which seems to us now so perfect?

But I have wandered far from the main purpose of this chapter, which was to survey the growth of the science of mechanics. A diagram of the sort drawn in Figure 18 displays the whole of the science of dynamics with the exception of the law of gravitation, which in its most qualitative aspect may be expressed by the statement that the space-time paths of two bodies will be curved toward one another, and the curvature will be greater the closer their approach to one another. If our diagram included the paths of several bodies as in the space-time map of the solar system which I presented in Figure 13 of the last chapter, all of these paths together may be said to produce a "field" which determines at each point the curvature of each path. Now let us introduce into the picture the path of another body called a test-body, which, merely for simplicity, we shall consider a comet, or any other body of sufficiently small mass so that its presence will not materially alter the

other existing paths. Now we may generalize the law of Galileo, that two bodies starting together always remain together as they fall, by stating that if the paths of any two test-bodies are tangent at one point they must coincide throughout their whole extent. This extremely simple theorem, although it still gives no quantitative law of gravitation, nevertheless already contains a considerable part of the remarkable gravitation theory of Einstein.

The theorem which we have just announced may be deduced from another which is even more general, namely, If in a given field a path passes through two neighboring points, no other path can pass through those two points without coinciding completely with the first. In other words, this path is a line which is *uniquely* determined by the two points. Now you will remember that in the second chapter we chose to regard the line uniquely determined by two points as the straight line, and we are immediately arrested by the thought that perhaps these paths in a gravitational field may be the straight lines of some new geometry, in the sense that a great circle is the straight line in the geometry of navigation. This thought has in



fact been adopted in the more recent development of Einstein's gravitational theory.

At first sight the idea seems preposterous. How can the helices in a space-time map of the solar system, by any stretch of the imagination, or by any elasticity in our use of language, be called straight lines? But did you ever try to draw a straight line upon a relief map? You will say that that cannot be done. But you might at least draw a line which seemed a little straighter than any other, and if you object to calling it a straight line you might be willing to call it the straightest line. We only know by experiment that measurements with a meterstick are fitted by Euclidean geometry, or that measurements with metersticks and clocks are fitted by our larger but still flat non-Euclidean geometry. How do we know that in the neighborhood of a massive body our measurements of position and time will conform to any simple geometry? The new theory claims that they do not, but it sets up a more difficult geometry which departs more and more from the simple flat geometry as we approach nearer and nearer to a large mass. In this new geometry the path of every body is one of these "streets called straight"; or, in other words, it is a geometry

in which the helical space-time paths that we have discussed are the nearest possible approach to straight lines. Strange as this new geometry appears, it has been found to conform to experiment in the three crucial tests which have so far been applied to it.

As a navigator sailing farther and farther from home makes observations and measurements which tally less and less with the predictions of the plane geometry of Euclid, so it is assumed in the new theory of Einstein, which is now being generally accepted, that the whole phenomenon of gravitation can be interpreted through the assumption that the flat geometry which we have employed in the preceding chapter does not quite lend itself to measurements of space and time made in the immediate neighborhood of a massive object. The departure from "flatness" which must be assumed is, however, so extremely small that estimates of distance made by Euclidean methods near the surface of the earth would never be in error by more than one part in a thousand million, while near the surface of the sun they might amount to as much as two parts in a million.

It was Clifford<sup>3</sup> who first suggested that

<sup>3</sup> W. K. Clifford, *Mathematical Papers*.

material particles might be regarded as humps in space, like "little hills on a surface which is on the average flat," and this is perhaps as near as we can come to visualizing modern gravitational theory. Professor Swann<sup>4</sup> has shown in

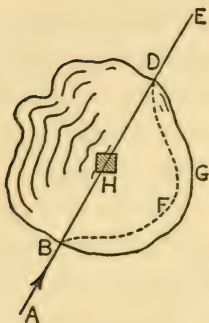


FIGURE 20  
(From Swann,  
*Science*, 1925)

an amusing way the contrast between the new view that the path of a particle is as straight as is permitted by the curved geometry which holds in the neighborhood of a large mass, and the old view that there is a force which causes the acceleration of bodies in a gravitational field. He says: "Suppose that the figure (Figure 20) represents a crater with a house

H in the middle, and that a traveller sets out to go from A to E by the shortest path. He will not necessarily pursue the path ABHDE leading down to the bottom of the crater, and through the house, because that may be too long. Nor will he necessarily go by the path ABGDE, because that may be too long. By taking some such course as ABFDE crossing the crater part of

<sup>4</sup> Swann, *Science*, 61, 452 (1925).

the way down, it is possible he will find a path shorter than any other, and this is the path that he will take." He imagines now an observer watching the man's progress from an airplane. To him the crater looks flat and he wonders why the man goes so far out of his course, and finally forms the hypothesis that someone in the house is directing a hose at the intruder, forcing him to make a detour. More detailed observation leads to more complicated assumptions, and finally it is seen that all the difficulties are removed by assuming a crater.

Of course it will be noted that in this illustration it is assumed that there *is* a crater and that there is *not* a hose, whereas in the problem of gravitation it seems at present optional whether we shall choose to use a simple geometry and introduce the alien concept of a force, which moreover becomes more complex as our observations become more accurate, or whether we shall eliminate this alien concept and employ a complex geometry, which perhaps also will become still more complex with an increase in our experimental and observational knowledge. In other words, it is far too early to decide, even tentatively, that the one is illusion, the other reality. At present the advantage seems to be



decidedly on the side of the purely geometrical interpretation.

We shall, however, find it far safer to speak of a "curved" geometry, which simply means a geometry with rules of operation which are more like those used in the geometry of navigation than they are like those used in the geometry of Euclid, rather than to speak of a curved space. Let us not allow our old enemy, the ether, with his stresses and strains, to reënter our domain, during his term of banishment, under an assumed name.

In all our discussion of gravitation it is assumed that we are dealing with bodies which are not charged electrically. If we are dealing with two objects, let us say the size of matches, a single electron, extremely minute as it is, is sufficient, when added to each object, to overcome gravitation and change attraction into repulsion. The smallness of the gravitational as compared with the electrical effect, has led a number of physicists to wonder whether the whole of gravitation may not be a small residuum of electrical attraction and repulsion. It would require only minor changes in the accepted laws of electricity to account for such a residuum, and we know that some alteration



must in any case be made. The mass of a negative electron is very much less than the mass of a positive electron or proton, and since we believe that these masses are due to what we call the electric field about each electron, the one field cannot be merely a replica of the other field, only with sign reversed. Now if it could be shown that between the ratio of these two masses and the constant of gravitation there is some simple relation, it would afford strong evidence for the view that gravitation is a mere residuum of electrical effects. However, if we should be led by this kind or any other kind of evidence to support such a view, it would presumably not lead to the demolition but rather to the amplification of the structure of modern gravitational theory.

Sometimes the scientist engaged in painting his picture of nature achieves, by a few bold and happy strokes of the brush, an effect so lifelike and beautiful that we are tempted to exclaim, "Do not touch it again, it is perfect." So the masterstrokes of Newton gave to mechanics the appearance of such completeness that many of his followers cried: "This is the whole of science. Given the motion and the masses of all the particles in the universe, all past and future

events will be spread out before our view." But no philosophy has been more barren than the mechanistic philosophy which this assumption engendered. To-day in the admiration and enthusiasm which have been so justly aroused by the remarkable achievements of Einstein a similar tendency is noticeable. As we read discussions of the new "world-geometry" we might be led to the notion that in this geometry lies the whole of science. But it is only another stage in the constant competition between those who are complicating science by newer and bolder experiments and those who are simplifying it by inventing more general theories and more comprehensive mathematics.

*Light and the Quantum*

Johnson, to Boswell: "Sir, to leave things out of a book, merely because people tell you they will not be believed, is meanness."

ONE of the oldest dilemmas of philosophy is that between continuity and discontinuity, involving also the much discussed question of action at a distance. I believe it was Carlyle who, granting that matter acts only where it is, asked, "But where the devil is it?" Where matter is and where it acts, and whether it is to be regarded as a continuum or a discontinuum, are questions concerning which the scientific mind has changed in recent years, and probably will change still further in the near future.

Although the idea that matter might be composed of indivisible particles had long been current in philosophy and in common speech, physics and chemistry up to the time of Dalton had been developed in terms of the continuum. Indeed, it is within our own lifetime that the atom and the molecule, from being mere working hypotheses, have come into a high degree of reality. Yet even now if it is asked whether the

world is continuous or discontinuous it is more than we can answer.

Put more modestly, the question is: Can the body of facts which are at our disposal be embraced in a set of principles which suggest the properties of an ideal continuum, or shall we select a set of principles based upon a discontinuum, or finally must we adopt one method for one set of facts and the other for another? Shall we use a continuous mathematics, such as geometry, or a discontinuous mathematics, such as the theory of integral numbers and rational fractions?

Perhaps the answer is suggested by our discussion in the first chapter, where we saw that in the process of growing thought the integers, once invented, began, as it were, to spin about themselves a continuous web until they became mere singular points embedded in the continuum of the general number series. A discontinuum includes only discontinua, but the continuum includes the discontinuum within itself. Being therefore the more general, it would seem that the continuum must continue to remain as a background of our scientific thought.

The science of hydrodynamics presupposes a fluid continuum, and no experiments on the

flow of liquids have yet shown any discernible deviation from its laws. Yet we know that water is only an aggregation of molecules, that such an idea as that of constant density must be invalid, and that if our vision could be amplified and our apparatus could be made more delicate, we should make observations not at all in accord with the theorems of hydrodynamics. Still it ought to be possible to enlarge the old science of hydrodynamics so as to include molecular motions, and among the theorems of this larger science the former theorems would still appear as important corollaries for the limiting case of an infinite number of molecules. Before this could be done it would be necessary to decide a number of questions; for example, whether, when two molecules collide, the impact is instantaneous or lasts a certain definite time, or whether it begins at infinite distance, becoming appreciable only at close approach.

We have made a little closer approach to the solution of such problems in the science of electricity, where that which was once called the electric fluid is now regarded as an assemblage of electrons. The electric field invented by Maxwell has been adopted into electron theory, so that we regard each electron as producing a



continuous field, the more important part of which is comprised within a small radius, but which does not become zero at any finite distance. So the mass of the electron, which is supposed to reside in the field, is not all concentrated near the center, but could only be comprised within a sphere of infinite radius.

By historical accident the charge of the electron is usually assumed to reside entirely at or near the center, but we should probably have a more consistent picture if this too were regarded as continuously distributed, so that the center of the electron would have no properties left except the purely spatial one of being the central point in this infinitely extended field. Thus each electron exists everywhere. I am afraid that if such a notion had been advanced, not by scientists, but by philosophers, it would have been characterized as pure mysticism, but after all is not this picture of electrons as mere points of discontinuity embedded in the continuum of their fields remarkably like our view of the integers as singular points in the continuum of numbers?

The atom and the molecule and the electron were therefore well on their way toward adoption into the old order of things, when Planck

advanced the *quantum theory* which, together with a large number of new experimental observations which followed in its train, indicated the existence of a degree of atomicity in nature far beyond anything yet conceived. Einstein, reviving the corpuscular theory of light, discovered a very important law concerning the interaction of light and electrons. Bohr proposed the theory that atoms change from one configuration to another *per saltum*, and thus found a marvelous and unexpected simplicity in the complex science of spectroscopy.

All this suggests a geometry in which distance is not measured but counted,<sup>1</sup> and a kinematics in which time ceases to have continuous extension, as though the mechanism of a clock were to vanish, leaving only the ticks! It is hard to predict how far we shall travel in this direction. It is my belief that further discontinuities will be discovered in the physical world, that have not yet been suspected. Nevertheless, it seems likely, on account of the greater generality and inclusiveness of the continuum, which I have already mentioned, that we shall return

<sup>1</sup> Dedekind believes that not even Euclidean geometry necessarily implies a continuum.

eventually to the continuous types of physics and mathematics.

In the meantime the quantum theory has introduced an element of paradox, the crux of which appears in the theory of light. Newton

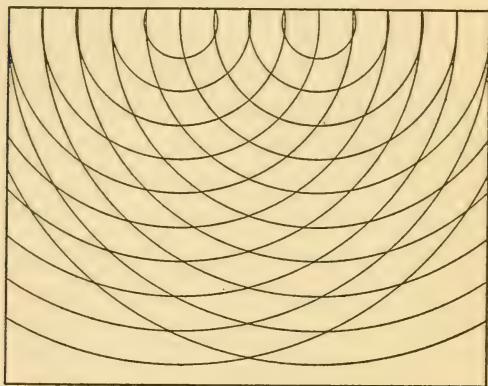


FIGURE 21

#### Diagrammatic Representation of Interference

thought of the transmission of light as having two aspects, in one of which it resembled the discharge of a bullet, in the other the spreading of a ripple or wave. It was the latter view which took root, for we know of nothing except wave motion which gives any analogy to the phenomenon of the interference of light.

If ripples or waves proceed from two points

on the surface of a liquid they meet to form a beautiful pattern, such as you must frequently have observed, and in which the ripples from the two sources alternately offset and augment

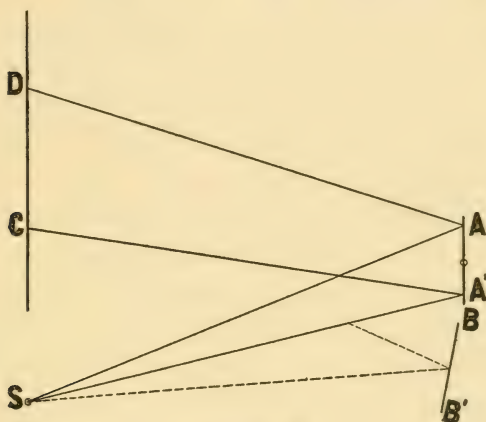


FIGURE 22  
Fresnel Mirrors

one another. Such a pattern may be represented diagrammatically, as in Figure 21, by drawing a series of concentric circles about two distinct centers. So if a beam of light is divided, and the two parts are allowed to converge once more, they produce a quite similar interference pattern.

Figure 22 represents one of the simplest con-

trivances of this sort. Light from a source S, let us say an incandescent filament, is reflected from a mirror AA' and falls upon the screen CD. But the light from the same source falling upon the mirror BB', which makes a slight angle with AA', also falls in the same region of the screen, where it alternately diminishes and enhances the light from the other mirror, thus producing upon the screen a series of dark and bright bands.

This is but one of numerous phenomena which are so varied in their manifestation that we must have some sympathy for the student who wrote in his examination, "The interference of light, as I understand it, is very little understood." Yet all these phenomena follow so directly from the undulatory theory of light, and have been reduced to such simple principles, and can be predicted with such accuracy, that there is no body of established thought which we should abandon with greater reluctance than the theory that light is in some way associated with some sort of waves or periodic disturbances proceeding outward in spherical symmetry from the source of light.

The older theory that these waves were mechanical disturbances in an elastic ether has,



however, been entirely abandoned in favor of Maxwell's view that they are periodic disturbances in electric and magnetic fields. It is a most remarkable fact that this great revolution in physical thought was accomplished without abandoning a single one of the set of mathematical equations derived from the old theory of optics. Does this not suggest that we may have still another revolution in our physical concepts and again retain the whole mathematical theory of optics? Yet it would seem that any new theory must begin with the assumption of some periodic phenomenon spreading outward in all directions from the emitting source.

Now comes, however, the revival of the corpuscular theory of light. I cannot describe here the beautiful experiments which corroborate Einstein's surmise that the energy of radiation consists of quanta or discrete corpuscles. Indeed there seems to be enough experimental evidence to permit us to announce two separate laws for the process of light. They seem to be mutually contradictory, and yet if they are both correct deductions from accurate observation the paradox must be due to our retaining too tenaciously some doctrine which has outlived its usefulness. Our task is therefore to seek some new

mode of thought in which the two following laws will seem mutually harmonious.

1. An atom possessing more than the minimum amount of energy may suddenly lose its superfluous energy, which travels as though it were a single corpuscle along a definite path with the velocity of light. The path is a straight line except where it is deflected in the immediate neighborhood of some material object, and in such deflection both the corpuscle of light and the other body obey the accepted laws of mechanics.

2. A single corpuscle or quantum of emitted light is subject to the laws of interference, or, in other words, the interference is not due to the interaction of a number of such corpuscles. Thus if the light from a certain source produces dark and light interference bands on a photographic plate, and this source is now replaced by a very faint source from which corpuscles are emitted only rarely, yet these corpuscles will strike the plate only where the bright bands were formerly seen.

If the phenomenon of light consisted of the emission of energy in corpuscles and nothing else, then in such an apparatus as shown in Figure 22 a corpuscle would be reflected from

either mirror regardless of the presence of the other mirror; the phenomenon of interference would not exist. It therefore seems necessary to return to the original idea of Newton that light has two aspects. Recognizing that the energy goes out in corpuscles, we must assume something else, something, indeed, which obeys the laws of wave motion, to determine where these corpuscles shall go. This something we may call the interference field, and it may be that we shall conclude that this field is present even at times when there is no actual emission of energy.

This is a view which has been advanced by Slater<sup>2</sup> and by Swann,<sup>3</sup> and the resolution of the phenomenon of light into its two aspects, one of which concerns the establishment of an interference field, and the other the actual transfer of energy, not only promises to be extremely useful, but I think it will prove necessary. But the particular way in which these two authors assume the light corpuscles to be directed by the field presents such difficulties that we are left with the feeling that the paradox of quantum theory is only partially resolved. If I now devote the remainder of this chapter to an

<sup>2</sup> Slater, *Nature*, 113, 307 (1924).

<sup>3</sup> Swann, *Science*, 61, 425 (1925).

attempt of my own to solve this puzzling question, I must apologize for presenting something to you which is only now in course of publication<sup>4</sup> and therefore has not run the gauntlet of scientific criticism. It may prove to be thoroughly erroneous. Often a "watcher of the skies" beholds some brilliant nova which later proves to be only a candle on a neighboring hillside.

Nevertheless, the issues which I wish to present have a very direct bearing upon the concept of time, which we are making almost the central objective of our discussion, and I believe our freedom of thought with respect to this concept will be enlarged by their presentation, even though they later prove to be inadequate. You will find what I say extremely repugnant to common sense, and therefore I am going first of all to remind you how greatly our notion of time has already changed in recent years, by bringing to your attention one of the most spectacular consequences of the special theory of relativity which I have postponed from the third chapter. This consequence,

<sup>4</sup> This work has now been published: *Proceedings National Academy of Sciences*, 12, 22 (1926); *Nature*, 117, 236 (1926).



bizarre as it still seems to the layman, has now been accepted into orthodox physics.

We may suppose that some years hence a few daring spirits looking for more worlds to conquer, will build a rocket-like machine, propelled by the expulsion of high-speed particles, and steered by directing the stream of particles one way or another. As their machine becomes perfected the inventors will find that they are able to leave the earth's atmosphere and visit the moon and nearer planets. In the meantime there is discovered a new kind of "fuel" which emits particles of far greater speed than any known before, so that one bold inventor believes that if he takes a quantity of fuel large compared with the bulk of his machine he may be carried far beyond the bounds of the solar system.

He makes the attempt, and by using the greater part of his fuel in one initial spurt he attains a speed within one-half of one per cent of the velocity of light. I shall not attempt to describe his sensations during this interval, but after he recovers and shuts off his accelerator he finds the most surprising change in the appearance of the heavens. From the rear window everything has disappeared, even the sun which near the beginning of the acceleration still



shone as a faint red disk; but through the front window he sees a dazzling array of stars of a brilliant blue color. Through the side windows the constellations come a little nearer to their customary appearance, and he sets himself to work upon daily astronomical measurements which prove so fascinating that five years pass in this pursuit. Suddenly he is alarmed, for his calculations show that in these five years he has proceeded to a point which seems to be fifty light-years away from the earth. His several chronometers of different types are all in agreement; he has eaten his three meals a day and slept eight hours out of every twenty-four. He is sure there can be no mistake, yet nevertheless decides to return to the earth, and, using the greater part of his remaining fuel, he reverses his motion and proceeds homeward with the same speed as before.

At the end of another five years he therefore approaches the earth, and using his remaining fuel to retard his motion, he reaches the earth's atmosphere and parachutes down to a place near his former home. Just ten years ago, according to his chronometers, the newspapers were full of his daring exploit and of various predictions as to the likelihood of his return,

and now he expects his arrival to produce an unparalleled sensation. But he finds everything changed; the people hardly understand him; and only after many bewildered questions does he realize that he is a second Rip van Winkle, and that the day of his return is not a decade but a century later than the day of his departure.

You will say that this is pure romancing, and surely there are grave doubts as to the feasibility of such a machine as I suggest; but, granting such a machine, no one who is acquainted with the elementary theory of relativity will deny that during the traveler's ten years' journey a century will have elapsed on the earth. I am simply putting into a more startling form the conclusion reached in the third chapter that there is no such thing as an absolute measure of the flow of time. In doing so it is with the thought that those who have made this great stride away from the traditional notion of time may not be unwilling to go a little further in the direction that I am about to propose.

We have seen how kinematics has been completely correlated with a pure geometry in which there is no intrinsic difference between up and down, or past and future; and we have also

seen how near the greater science of mechanics is approaching to a similar state. Indeed, as we survey all the simple phenomena comprised within the science of physics (leaving the so-called irreversible processes for our next chapter) we find that it is only in certain phases of the subject related to the theory of radiation that any dissymmetry is assumed between past and future. Hitherto it has seemed impossible to picture the process of absorbing light as strictly the reverse of the process of emitting light. In the old theory we thought of emission as a series of concentric waves emanating from the emitting body, but we did not consider the absorption of light as associated with a series of concentric waves converging upon the absorbing body. If we can now devise a theory in which emission and absorption are quite symmetrical it will greatly simplify our concept of physical time.

When we consider once more the simple apparatus of Figure 22, where C now represents the center of a dark interference band, we see that if we cover the two mirrors, and then uncover the mirror AA' for an instant, a corpuscle of light may proceed from S to A' and from A' to C; but if we uncover both mirrors for the

same instant no corpuscle of light can reach that spot. As we meditate upon this mysterious phenomenon there constantly recurs to us the scarcely credible thought that in some way an atom in the source can foretell before it emits its corpuscle of light whether one or both of the mirrors are going to be uncovered. Ridiculous as it may sound, this is essentially the theory that I am going to offer.

It is generally assumed that a radiating body emits energy quite regardless of any near or distant objects which may ultimately absorb it. In other words, it is said to radiate "into space." I shall make the contrary assumption that an atom never emits light except to another atom, and in this process of exchange the two atoms play quite coördinate rôles, so that one can no longer be regarded as an active agent and the other as an accidental and passive recipient.

I do not wish to minimize the conflict between this view and that of common sense. The light from a distant star is absorbed by a molecule of chlorophyl which has recently been produced in a living plant. We say that the light from the star was on its way toward us a thousand years ago. What rapport can there be between the



emitting source and the newly made molecule of chlorophyl? Again, let us suppose this same star to be the source of light in Figure 22. By uncovering both mirrors we prevent a particle of light from reaching the point C. Do we thus prevent its original emission in just this direction? If so it means that we can, in a trivial way, but nevertheless in principle, affect the course of what we call past events.

Offensive as such an idea is to all our established notions of causality and temporal sequence, we must remember that these notions have arisen from the observation of processes which are very different from the elementary reversible processes which we are now considering. Unless the result of some actual fact of experiment or observation can be brought against the new view, we need not be deterred by this conflict with common notions, especially since we shall see that by this means certain inconsistencies between prevailing physical ideas and the geometry which so admirably interprets the kinematics of relativity are removed.

We admit that a radioactive substance emits its particles in all directions without regard to their later absorption by other matter, and why



should we make a different assumption regarding light? The answer is suggested by the new geometry, which I recall to your mind through Figure 23. We remember that there is a sharp distinction between all the space-like lines,  $OX$  and  $OX'$ , and the time-like lines such as  $OT$  and  $OT'$ . Between the two classes are the singular lines  $OL$  and  $OL'$ , which belong to neither of these two classes and bear no more resemblance to one class than to the other. The path of a material particle in space-time is always one of the time-like lines such as  $OT'$ , and the slope of this line with respect to a chosen time line  $OT$  represents the velocity of the particle. On the other hand, the slope of a space-like line, such as  $OX'$ , in no way suggests a velocity of matter or energy. Now in spite of the symmetry demanded by our geometry, we affiliate the line  $OL$  with the time-like lines and say that it represents the velocity of light. This concession to traditional thought certainly does violence to the geometry, and if we continue to make such concessions it must be with the full realization of the unique character of the process of radiation.

Let me recall another remarkable feature of the relativity geometry. Such intercepts of sin-

gular lines as  $OL$  and  $OQ$  are said to be of zero length. This is an idea of which great use has been made in the mathematics but none in the physics of relativity. The proposal that I am

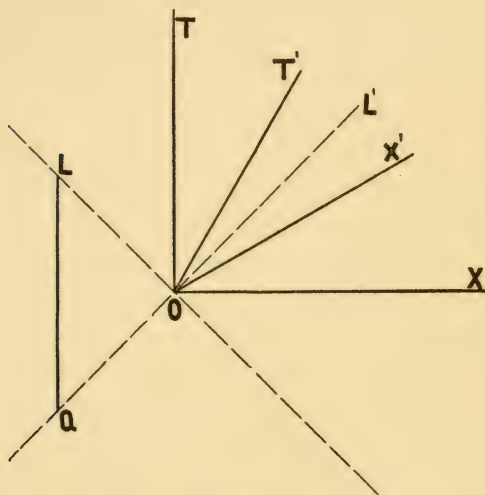


FIGURE 23

making is tantamount to assuming that such a distance is also zero in a physical sense, so that two atoms, whose loci are  $OT$  and  $QL$ , may be said to be in *virtual contact* at any two points such as  $O$  and  $L$  or  $O$  and  $Q$ , which are connected by singular lines. If the point  $O$

represents the event of the emission of a light particle by the one atom, and if  $L$  represents the event of the receipt of the particle by the other atom, these two events may, in our ordinary way of speaking, be thousands of years apart, but any such statement depends upon an arbitrary choice of an axis of time. If in our figure we take, not  $OT'$ , but other time axes lying nearer and nearer to  $OL$ , not only the time elapsing between  $O$  and  $L$ , but also their spatial distance, approaches zero, according to Einstein's principles.

I have spoken of virtual contact as distinguished from ordinary contact, but I do not wish to imply that it is any less a physical contact in the one case than in the other. I claim that my eye touches a star as truly as my finger touches this table. When two atoms are in ordinary contact we do not inquire how one ascertains that the other is in a position to receive energy, nor need we so inquire in the case of virtual contact. It will suffice to discover and to describe the conditions of relative orientation between the atoms, and the like, that permit the exchange of energy. These, however, are technical questions, and I cannot discuss here the methods by which the new idea may be shown

to lead to the various equations of optics, nor how the undulatory field, of which I have spoken, proves to be only a sort of shadow, cast upon space, of periodic changes occurring in the emitting and absorbing atoms themselves.

I shall, however, just mention a crucial experiment which, though difficult, seems to be experimentally feasible. In the apparatus shown in Figure 22 let us suppose that C is the center of a dark interference band and D is the center of the next bright band. According to all former theories, the light falling upon the mirror AA' produces thereon a uniform pressure; but according to the new theory no corpuscles are striking the end A' while many strike the end A. The mirror therefore, if supported by a fine suspension, would tend to rotate about its center.

If such a rotation can be observed it will constitute powerful evidence for a theory which not only is in full harmony with our relativity geometry, but also removes the last foothold in the physical sciences of the concept of temporal causality.

## VI

### *Probability and Entropy*

**I**N this chapter I am going to speak of the application of a kind of mathematics which, while based on arithmetic, occupies an independent position of great significance. It is a subject which should be taught in every elementary school, but the average educated man has no knowledge of it except that which he may have derived from a practice which society regards as vicious. I refer to the theory of probability, of which the elementary principles are of daily applicability.

You may say, "I am not a betting man"; but have you not registered a bet with the insurance company that your automobile will be stolen this year? When you double your opponent's bid at bridge are you not taking a chance? Every decision in life is a gamble, frequently involving highly complex features, but it is in the midst of such complexities that the elementary methods of probability may often be successfully invoked.

Sometime perhaps the science of logic will not regard every statement as true or false, but



each as possessing a certain estimated degree of probability; and the result of a generalized syllogism will be a numerical fraction representing in its turn a probability.<sup>1</sup> Let us suppose that a certain statement has but one chance in a million of proving false.<sup>2</sup> Suppose now that we have a million such statements, what is the likelihood that all of these statements will together prove to be true when tested? It is less than an even chance.

A chemist who needs to know the density of mercury at its boiling point finds in the literature records of half a dozen determinations. Sometimes he is satisfied to take their average, but ordinarily he scrutinizes the methods which have been employed and estimates the reliability of each investigator. He thus gives a different weight to each determination on the basis of its relative value, and in assigning these several weights he is not using mathematics but common sense. It is only after this part of his task

<sup>1</sup> See Charles S. Peirce, *Popular Science Monthly*, 1878.

<sup>2</sup> This is a very high degree of probability; we feel sure that the sun will rise to-morrow, but during the whole of written history the sun has risen and set less than a million times. Of course, owing to our knowledge of astronomy, we have other reasons for believing that the sun will rise, in addition to the fact that it has always done so.

is completed that he then proceeds by purely mathematical methods to ascertain the most probable value of the density of mercury and the limits of its probable error.

The theory of probability deals with data which it does not itself provide, but which are taken from without. As the science of geometry tells nothing about the dimensions of this room, but calculates diagonal distance after the length and breadth are given; so, if in tossing a coin the chance of a head is one in two, the chance of getting all heads in three independent throws is calculated to be one in eight. But the theory of probability does not attempt to prove that the chance of throwing a head is exactly equal to the chance of throwing a tail; and obviously it is not, for even if we leave aside the mechanism of tossing, the two faces of the coin are not mechanically identical. Nor can dice be made so perfect that each one is not "loaded" in some slight measure.


While we never can draw a perfectly straight line, the idea of a straight line is essential to geometry. So we never actually meet an entirely level chance. Having imagined an ideal coin with two sides which are quite alike mechanically, we must also invent an ideal tossing

mechanism. We do not demand precision in such a machine. It might, indeed, be so nicely tooled and so uniformly operated that a coin placed head-up in the machine would always land head-up. This is the very opposite of what we desire; we must introduce a concept alien to mechanics and in a sense hostile to mechanical concepts,—the idea of randomness.

The level chance, the random distribution, suggest in many ways the notion of straightness in a line or evenness in a plane. Indeed, we might emphasize this correlation by defining a straight line in the following peculiar manner. Suppose that a room contains a number of objects moving about in a random way. If now between two fixed points in the room we consider an imaginary line drawn, this line will from time to time pass through one of the moving objects, and we may count the number of these intersections over a long period of time. The line between the two fixed points which cuts the smallest number of objects during this long interval of time is the straight line. However, if the objects were not in absolutely random motion, but were, owing to gravity, for example, more frequently in the lower part of the room than in the upper, then the line cutting the least

number of objects would in general be a curve. A straight line and an even chance are assumed when there seems to be no "sufficient reason" for a more complicated assumption.

I cannot here describe in detail the theory of probability and its many useful applications, but must content myself with a brief discussion of its application to one science which, because it stands apart from all others, is of great philosophical interest. The sciences of geometry, statics, dynamics, electromagnetics, all assume a knowledge in full detail of the system under investigation. The science of thermodynamics, on the other hand, ignores all detail. We are given a box of unknown contents with two wires protruding, and are told that there is a certain voltage between these two wires. We are also told that when a certain current is drawn from the wires a certain amount of heat is given off by the box. From these factors alone we calculate what the voltage between the wires will be when the temperature of the room changes. A person who has known only the sciences of detail that I have mentioned, and later realizes the powerful support which he may obtain from this science which knows no detail, is like a swimmer who learns various strokes by which





he can swim ten yards or a hundred yards without drowning and only later becomes aware that without swimming at all he can float, buoyed up by the water itself.

The validity of thermodynamics is independent of mechanics. In fact, the science of mechanics is not altogether satisfactory; a study of the specific heats of substances has shown that it is necessary to make some change in the assumptions of mechanics, but we do not yet know what change must be made. Yet even if we had a science of mechanics so complete that it would not require alteration for another century; even if we could solve the problem of two bodies or three bodies or a hundred bodies; what would we do when we came to a million? The mere description of the initial state would occupy us for a lifetime. The mechanical method does not *reach* far enough. We may measure the dimensions of our garden with a footrule, but we do not try by the same method to find the distance of the stars.

It is the field of the highly complex that thermodynamics claims as its own. This science is based upon two fundamental principles, of which the first deals with the conservation of energy. It would be interesting to trace the his-



tory of the concept of energy; the curious and almost tricky invention of potential and latent energy; how once it was tried to make the concept of energy supreme in physics, thrusting matter into a secondary place; how the principle of relativity, clarifying so much that was obscure, shows that energy content and mass are merely different ways of stating the same thing, as we express a distance in miles or in meters; how, finally, energy itself is now regarded as only an arbitrary cross section of a greater entity, the tensor.

However, it is of the second law of thermodynamics that I wish to speak, and those irreversible processes which are known as processes of degradation or of dissipation. A body falls to the ground; it may rebound once or twice, but comes eventually to rest; when hot and cold bodies are brought together their temperature is equalized; gases mix by diffusion and then it is a difficult matter to separate them; in all such processes there seems to be a general running down, and as a measure of the run-downness of things Clausius coined the name *entropy*, and announced the general law, "The entropy of the world tends toward a maximum." Each of these running-down processes is said to

be irreversible; not that its initial condition cannot be restored, but if it is, it is always at the expense of something else that is running further down. In technical language, the entropy of one system may become less, but only if that of other systems increases by at least as much.

Thus all things are supposed to be moving toward a final state of equilibrium where all happenings cease. Even though it is not claimed that this changeless Nirvana would ever be reached in finite time, this picture of a universe moving toward old age and ultimate death has seemed to many a gloomy one. I am therefore glad to express the belief to which I have recently come, that there is no evidence for the hypothesis of continual degradation.

Astronomy seems to show no cosmic running-down. Some stars fade but others grow brighter. The sun is constantly pouring upon the earth a great flood of energy and apparently without recompense. The source of this energy is still a complete mystery, but there is no evidence that it is giving out. Many geologists now believe that as far back as the history of the earth's crust can be traced, perhaps a thousand million years, during which great climatic fluc-

tuations have occurred, there is nothing to show that the earth has grown colder on the average.

In any case this unidirectional flow of energy from the sun without apparent source concerns the law of conservation of energy rather than the law of dissipation. If we have to state the latter in conformity with scientific usage, we must not talk of the universe of which we know so little, but of some isolated system which can be studied at will. Thus the law of Clausius must read, "Any system left to itself approaches in a single direction a definite state of equilibrium, or, in other words, its entropy increases steadily toward a maximum." It is this statement that must be challenged.

However, before proceeding with this analysis, I must point out the great influence of the unidirectional process that we are now considering upon the concept of time. In the pure geometry of space-time there is no dissymmetry between up and down, or between forward and backward in time; nor have we found in the laws of mechanics, to which that geometry is so admirably fitted, any evidence of such dissymmetry. Even our new picture of the phenomenon of radiation gives no intimation of unidirectional time. On the other hand, all of the inci-

dents of daily life give us quite a different view. As I remarked before, nothing is more ludicrous than a moving picture run backward, and this illustrates the extreme dissymmetry of past and future in actual life, which is full of abrupt occurrences in which complex structures, patiently evolved, meet with instant dissolution. A forest burns; a shell explodes, and of all its elaborately prepared chemicals and its intricate machinery only formless fragments remain; a man dies and thus brings to an abrupt end a development which has been proceeding, not merely through a lifetime, but through the ages.

Here are phenomena so entirely unlike the simple reversible processes we have considered hitherto that we are tempted to believe that our idea of unidirectional time is due to the existence of irreversible phenomena. This thought was, I believe, first expressed by Professor W. S. Franklin,<sup>3</sup> and it is one which immediately gives us a deeper insight into the meaning of time. It leads us to split the time-concept into two quite distinct ideas, as Bergson<sup>4</sup> has done in

<sup>3</sup> W. S. Franklin, "Entropy and Time," *Physical Review*, 30, 776 (1910).

<sup>4</sup> Bergson, *Time and Free Will; Creative Evolution*.



his illuminating essays. The one part, which we may call one-way time, is that unidirectional sequence which we find in our own consciousness and memory, and also in the world of apparently irreversible phenomena. The other part, which we may call two-way time, is one in which future and past are symmetrical. It is purely and simply one of the dimensions in the space-time manifold of kinematics.

This distinction, however, means that for one kind of physics, mechanics, we must use one of these concepts, and for another branch of physics, thermodynamics, we must use the other. I propose to show that for all physical science only one time-concept is necessary, namely, the symmetrical or two-way time. But before doing so let us proceed a little further with the discussion of the second law of thermodynamics.

Do you believe in miracles? Let us consider a box with a one-gram weight resting on its floor. Let us place this box in a bath maintained at an extremely constant temperature, we will say 65° F., and let the whole be protected by the most perfect mechanism that we can think of to shield it from external jars. Let us, in other words, shut it off from all external influences, leaving only a small hole through which we may



observe the weight. We may look into the box millions of times and always find the weight upon the floor, and we then state this to be a law of nature. But the time will come when we look in and find the weight some distance from the floor. It will not happen often, but we can calculate with a high degree of accuracy the chance of finding it, let us say, ten centimeters or more from the floor. This chance is so very small that I cannot express it in any ordinary way. We state chances as fractions, but to denote this chance I should have to put down a decimal point and zero after zero, and would spend my whole lifetime before I could write down a number not a zero. But the calculation is none the less exact.

The chance becomes larger if I consider smaller weights and lesser heights from the floor. Let the height be one hundred million times as small and the weight also one hundred million times as small, and then the calculation shows that if we look in every second we shall find the weight as far off the floor as this 6.32 times in every million years. If you bet five to one on the appearance of this phenomenon in a million years you might lose at first, but would come out ahead in the long run.

If we take a still smaller weight the chance goes up very rapidly, and if we examine particles which are just visible with a good microscope we find them hopping about in what is known as the Brownian movement, and which we now know to be identical with that thermal agitation which all molecules or groups of molecules exhibit, and which always increases with increasing temperature. From a study of these movements Perrin was able to draw some extremely important deductions with the aid of the same formula as I have just employed.

If the jumping of the one-gram weight was a miracle, here in the Brownian movement thousands of such miracles are constantly occurring before our eyes. It is not true that things left to themselves approach a constant state, but only that they approach a state which ordinarily appears constant to us because of the dullness of our perceptions.

It was Maxwell who made the ingenious suggestion of a little demon who could see and distinguish between the individual molecules. Let us consider, as a type of irreversible phenomenon, the mixing of two gases. A container with a partition in the middle has oxygen on one side, nitrogen on the other. If a small hole is

made in the partition the gases mix by diffusion, so that ultimately we cannot distinguish between the contents of the two sides. This, according to the ideas of classical thermodynamics, was a completely irreversible phenomenon. But suppose the demon stationed at the orifice with a small shutter which he can open or close at will, and suppose that he decides to let only oxygen molecules pass in one direction and only nitrogen molecules in the other. Can he not through this exercise of *conscious choice* ultimately restore the original condition, with all the oxygen on the one side and all the nitrogen on the other? That such a reversal of a so-called irreversible process may occur, even without a demoniacal agency, was first recognized by Willard Gibbs, who announced that such a reversal is not an impossibility but only an improbability; and this idea, which was thoroughly developed by Boltzmann a generation ago, has grown to be an important, and almost the most important, guiding principle of modern physics. Its full utilization has been delayed owing to misconceptions as to the nature of light, but it seems to me that these difficulties are now removed.

In order to illustrate the modern statistical

view of thermodynamics, let us consider a fresh pack of cards, all sorted, for example, ace, king, queen, etc., of spades, ace, king, queen, etc., of clubs, and so on. Such a grouping of the cards is easily described and easily discernible. If I spread the pack before you, one-half is obviously black, the other red. If I should put this pack into some sort of shuffling and dealing machine the first deals might give some very singular hands, but as the process continued a point would be reached where we would say that the pack was well shuffled. Now any one of the particular arrangements obtained by shuffling is just as improbable *a priori* as the first well-sorted arrangement, and by this we mean that if we note the arrangement, card by card, in a well shuffled pack, this identical arrangement is no more likely to recur than the arrangement of the fresh pack. Nevertheless, while the number of possible well-sorted or easily *describable* arrangements is very small, there is an enormous number of arrangements of a *nondescript* character. As the shuffling continues the arrangement constantly changes; there is no approach to any one particular arrangement, but on the other hand it rarely happens that a remarkably distinguished arrangement occurs. If one of you



ever picks up a bridge hand with thirteen trumps you will probably tell it to all your friends, and, indeed, such a hand is a very rare fluctuation from the general run of nondescript hands.

There is a psychological element in all this. If we look down from a window upon a street of New York and see the people emerging from a subway exit, some occurrence may draw our attention to one group of ten people. We make note of their faces and their clothes. They thus become in some measure familiar to us, and it strikes us as remarkable that these ten people are about to melt into the crowds of the great city and will never again meet together. Yet all through the day each will be one of a group which differs from the first only in that we have not become acquainted with its members.

Since the rules of chance arrangements contain every essential feature of the second law of thermodynamics, let me illustrate it further by this apparatus (Figure 24), in which I have one tube containing ten black balls and one tube containing ten white balls.<sup>5</sup> If I invert the

<sup>5</sup> For this illustration and several references in this lecture I am indebted to the interesting book of C. E. Guye, *Physico-Chemical Evolution*.



apparatus so that the balls are mixed, the first distribution or two may show evidence of the original sorting, but after one or two good mixings the balls enter the tubes in random arrangements which follow the simple laws of chance. As the mixing continues we do not get nearer and nearer to the even distribution of five blacks and five whites in each tube; on the contrary, the arrangement of six and four occurs more frequently than that of five and five, and occasionally greater *fluctuations* from the mean occur. Indeed, if we have patience to mix them two or three hundred thousand times, we shall probably come to the original well-sorted arrangement.

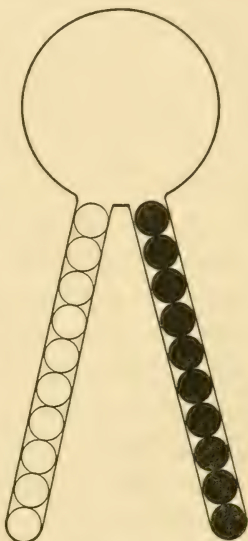


FIGURE 24

If we stop there and plot our records from the original complete sorting to the final complete sorting we shall obtain a zigzag curve which shows no dissymmetry on the average with respect to the beginning and the end. If instead

of the twenty balls I had taken a million grains of white and black sand the mixing would have appeared to be more thorough, but the fluctuations, although not so evident, would still be there, and if we should take an enormously greater number of particles we would have a complete analogy to the mixing of the molecules of oxygen and nitrogen. And here again we must assume the existence of fluctuations from the mean, sometimes small, sometimes great, and after the lapse of a vast period of time even so great as to give once more pure oxygen and pure nitrogen. Here again if there were a way of recording these fluctuations between the original distribution of pure oxygen and pure nitrogen until we arrive once more at the same distribution, there would be nothing in all these fluctuations which would not be symmetrical with respect to past and future.

Are all cases of irreversible phenomena as simple as this? For many years I have found this one of the most perplexing and tantalizing of scientific questions. As long as it was supposed that radiation, even from a single atom, goes out in a way which can never be entirely reversed, there seemed an insurmountable obstacle to the view that all irreversible processes are

mere mixings and shufflings, with no element of novelty except that which arises from the vast number of elementary processes involved. But if we now regard radiation also as a process of exchange of energy between two atoms, the emitting and absorbing atoms playing symmetrical parts, this obstacle is removed.

We then find not even a shred of truth left in the statement that an isolated system moves toward a state of equilibrium. It will move toward it and move away from it, and in the long run as often in one direction as in the other. It is only when we start far away from the state of equilibrium, that is, when we start with some state of unusual distinction, and when we follow the system a little way along its path, that we can state that it will, as a rule, proceed toward more *nondescript* states.

The second law of thermodynamics has come to be regarded as one of the most powerful and inexorable of nature's laws. It is beginning to dominate chemistry and the biological sciences, and its sway extends from the pure sciences into the various domains of technology. Now, however, that we have looked behind the scenes, it almost seems as though it were all a sham, like some silly clown garbed in the ermine of roy-

alty. Stripped of its finery we find that the second law states that if a pack of cards is thrown into a shuffling machine the chances are that it will become shuffled.

If this discovery comes to us as a great disillusionment it is only because our minds are tinged from infancy with the hoary superstition of the absolute. We say, "If this great law is not always true what becomes of our other exact laws?" But can we have no reverence for any institution without making the childish assumption of its infallibility? Can we not see that exact laws, like all the other ultimates and absolutes, are as fabulous as the crock of gold at the rainbow's end? We have a sense of contentment as we travel day by day through the beautiful and fertile lands into which we are led by one of these will-o'-the-wisps. It is only after someone cries, "I have caught it in my hands," that we experience the bitterness of disappointment.

Our respect for the great law of entropy need be no less because it has its exceptions, nor yet because it can be reduced to the obvious. Think of all the propositions of Euclidean geometry which can be reduced to the simple statement that there are parallel lines and that there are

circles. Familiarity breeds contempt only for artificiality and pretense; and the law of entropy becomes a greater law when limited, as we now must limit it, by the following statement: An isolated system always proceeds in the direction of greater entropy, provided that it be so far removed from the state of equilibrium that the chance fluctuations in that state may be ignored.

A law of nature becomes a better law when we can predict the exceptions to it. If someone tells you that the sun will continue to rise and set through all future time you are not deceived by the finality of this statement. You know that here and there through the galaxy of stars many cataclysms occur which would wipe out whole solar systems. If through a study of the average frequency of such occurrences, or through a study of the movements of all known bodies in the neighborhood of the sun we predict that the earth will be safe for another trillion years, we have far more information than is furnished by the trivial statement that the sun will go on rising for ever.

And now let me revert once again to the concept of time. If you agree with me that in mechanics, as illustrated by the solar system,



there is no distinction between forward and backward in time, that in the elementary processes occurring between atoms, even in the exchange of radiant energy, there is no dissymmetry of past and future, how can we believe in such dissymmetry in a so-called irreversible process? What mysterious element do we introduce when we proceed from two or three atoms to a million or more? We have seen that the irreversible process does not really exist and that the sense of mystery is only due to our unfamiliarity with large numbers. We may therefore conclude that in all physics, including thermodynamics, there is no need of introducing a distinction between past and future, nor any concept of time except what we have called two-way time, which may be regarded as only one mode of extension in our four-dimensional manifold of kinematics.

Whence then arises the extraordinary dissymmetry with respect to past and future which we meet in everyday life? For one thing we have a steady flow of energy from the sun which is, as it were, constantly throwing sorted decks of cards into our great shuffling machine, and as we watch them pass from the describable arrangements to the nondescript we have a sensa-

tion of steady flow such as a miller experiences as his wheel is turned by the constant current of a mountain stream. He does not ask where the water comes from. Nor do we yet know the source of this cosmic energy.

Is this the whole basis of our idea of one-way time? We have observed that the notion of a flow of time in one direction seems to be derived partly from the unidirectional sequence in our own consciousness, and partly from the extreme dissymmetry between past and future in the world about us. When we see a boat upon a calm sea we know the direction of its motion by the wake that it leaves behind. So our own thoughts leave a wake of memories, and here and there we seem to see glimpses of a similar phenomenon among other living creatures. Again, when we meditate upon those cataclysmic events which make it so difficult to think of time turned backward in its flight, we realize that not all, it is true, but certainly the most striking examples are furnished by the sudden demolition of highly complex structures slowly built up by man or by other living things. All this leads us to wonder whether in the world of animate nature there may not be phenomena which

transcend thermodynamics as thermodynamics transcends mechanics.

The thought that living beings may escape from the law of entropy we owe to Helmholtz, although many years earlier Kelvin<sup>6</sup> also had written with respect to the second law, "It is impossible by means of an inanimate material agency to derive mechanical effect. . . ." The few experiments which have so far been made with a view of finding an actual reduction of entropy in systems containing living organisms have been unsuccessful, but even if they continue to fail, yet when we observe the enormous complexity of animate forms it is hard to believe that they could be the result of a blind fortuitous shuffling of atoms.

Borel<sup>7</sup> makes the amusing supposition of a million monkeys allowed to play upon the keys of a million typewriters. What is the chance that this wanton activity should reproduce exactly all of the volumes which are contained in the library of the British Museum? It certainly is not a large chance, but it may be roughly calculated, and proves in fact to be

<sup>6</sup> W. Thomson, *Dynamical Theory of Heat*.

<sup>7</sup> Émile Borel, *Le Hasard, Elements of the Theory of Probability*.

considerably larger than the chance that a mixture of oxygen and nitrogen will separate into the two pure constituents. After we have learned to estimate such minute chances, and after we have overcome our fear of numbers which are very much larger or very much smaller than those ordinarily employed, we might proceed to calculate the chance of still more extraordinary occurrences, and even have the boldness to regard the living cell as a result of random arrangement and rearrangement of its atoms. However, we cannot but feel that this would be carrying extrapolation too far. This feeling is due not merely to a recognition of the enormous complexity of living tissue but to the conviction that the whole trend of life, the whole process of building up more and more diverse and complex structures, which we call evolution, is the very opposite of that which we might expect from the laws of chance.

Let us suppose that one of our shuffling machines deals a hand with four aces and the next time does the same thing. We say it is a remarkable coincidence, but if the machine keeps on making such deals we say it is not "straight." Our whole application of the theory of probability to thermodynamics has rested on



the assumption that there is random shuffling, and the great achievements of thermodynamics have seemed to justify this assumption. But these achievements have all been in the domain of physics and chemistry. After our experience with the geometries, where we found it necessary for the interpretation of some parts of mechanics to introduce a curved rather than a flat geometry, we must be prepared, as we increase the scope of application of probability theory, to find crooked shuffling machines as well as straight ones.<sup>8</sup>

Here we are beyond the help of mathematical calculations. But the observation of animate nature leads to the almost irresistible conviction that here emerges into our perception a new element, alien to the randomness that characterizes the physical sciences; indeed, that living creatures are cheats in the game of physics and chemistry. It seems that animate creatures

<sup>8</sup> Such crookedness must not be confused with the mere lack of a level chance. If there are a hundred white balls and a hundred black balls in a box, and these are drawn by a person blindfolded, we can easily calculate the chance that the first ten balls will be white. If there are a hundred white balls and a hundred and ten black ones the calculation is a little more difficult but can be made with equal exactness. If, however, the drawer recognizes a slight difference in feeling between white and black balls, then we refuse to make any calculation as to the result of the drawing.



alone are striving for distinction in the midst of the almost overwhelming leveling forces in the great democracy of the atoms. It is not necessary to assume that we shall find on any large scale an obvious exception to the law of entropy, for in the great flow of irreversible processes it is far easier to swim with the current than against it. But here and there it may be possible to take advantage of the eddies which we have called the fluctuations from the average state. These fluctuations are no more discernible to us than the waves of the ocean to an aviator; but living organisms run through a great range of size. If we care to make a very rough scale we may say that a man is to a fly as a fly to a microbe, as a microbe to the organism of a filtrable virus, or of the "phages" which are now interesting the bacteriologist, and finally as such an organism is to an organic molecule. If the more minute creatures, or the smaller subdivisions of our own organism, are capable of choosing between individual molecules or of utilizing those fluctuations which, although invisible to us, are great maelstroms to them, would it be more wonderful than it is to see a fowl selecting little grains of seed, or an

eagle soaring to ever greater heights by utilizing the slight variations of the wind?

It may have seemed strange that in outlining the contents of this work, which handle for the most part concepts with which I have some degree of acquaintance, I touch in the last chapters upon biological problems where I have only the merest smattering of information. However, I shall emphasize again in the next chapter how subtle the distinction is between animate and inanimate nature. Certainly a glance at the characteristics of living things cannot obscure and may perhaps clarify our understanding of some of the concepts of inorganic science.

## VII

### *The Non-Mathematical Sciences*

I HAVE devoted six of my eight chapters to sciences which are either wholly mathematical or which need mathematics for their understanding and their expression. And yet if I seem to have given them the greater weight, it is merely because they require only a few concepts and these of the most highly refined nature, as compared with the great sciences which derive but little assistance from mathematics, and rest upon independent foundations; certainly not because the attainments in these latter sciences are less worthy of our admiration. These non-mathematical sciences involve a very much larger number of separate concepts which, for the most part, show a little more of the raw material of experience from which they have been derived, with less refinement in the processes of abstraction and idealization. As an example of these let us consider the concept of material substance.

Matter has not been fully defined as a scientific concept. If we should say it is that which has mass, then light would be classed as matter.

The natural philosophers of a few generations ago spoke of "imponderable substances" such as heat, light and electricity, but all these are now known to possess some weight. The fact is that in the complex domain which we are now entering we cannot afford to be disturbed by such verbal imperfections; there are too many other things to think about. The science of dynamics grew out of the study of just one property of matter, its mass or weight, ignoring the hundreds of other properties which it exhibits. But we do not go very far toward describing a humming-bird when we say that it weighs ten grams.

In the great variety of properties that various materials exhibit it is necessary to look for some traits of permanence or identity. It is easy to discover that ice and water, despite their dissimilarity, have something in common. So also we discover that to the action of fire or acids a statue of Praxiteles behaves like the block of marble from which it was sculptured. It is these intrinsic properties, which are independent of external form and appearance, that serve for the classification of the different substances of which chemistry has analyzed and classified perhaps two hundred thousand, and

even these represent but a minute fraction of the number that could easily be prepared in the laboratory or found in nature. Out of this variety of chemical substances men have striven to obtain some simplicity or even unity. The Greek elements, earth, air, water and fire, the alchemists' elementary mercury, salt and sulphur, were crude efforts in this direction.

Robert Boyle, one of the great contemporaries of Newton, was the first to make a satisfactory classification of substances. He showed that out of combinations of a small number of elementary substances all of the numerous compound substances could be obtained, and this is the basic classification of chemistry to-day, although we see that it possesses no finality now that the elements themselves are being decomposed. The prophecy of Prout that all substances would ultimately prove to consist of nothing but hydrogen has been nearly fulfilled in the recent discovery that protons and electrons constitute the material universe.

We have learned to analyze, not only the substances within our reach, but the sun and stars, and there spectroscopy shows that, except for a very few lines of the spectrum which are still mysterious, they are composed of the same



elements with which we are familiar. In spite of the enormously greater reach of modern telescopes, the stellar universe is very much smaller than it formerly was, and we can imagine the modern child saying,

Twinkle, twinkle, little star,  
How I used to wonder what you are.

Indeed, after the discovery that just two kinds of building blocks suffice for our whole galaxy, it seems that greater restrictions have been placed upon speculative philosophy than ever before in the whole history of science.

But let us beware of too much complacency. The discovery that millions of compounds are composed of a hundred elements is now followed by the discovery that the hundred elements are composed of two sub-elements. Yet even if you and I and the stars are nothing but protons and electrons, this knowledge does not at present aid us much in solving the problem of the constitution of the proteins. Have we not seen in the last chapter how the great science of mechanics is helpless in the face of large numbers and must call to its aid the alien methods of thermodynamics? Many of the problems of chemistry for a long time to come must be

studied by methods which are not very different from those of the early alchemists.

After the work of Boyle the next great step in chemical classification came through a study of the phenomenon which we know as reduction and oxidation, but which was first called phlogistication and dephlogistication; and although I have recently shown<sup>1</sup> that this classification is not absolute but only convenient, it remains and will remain one of the leading ideas in chemistry. The phlogistonists were not content with the idea alone, but must add a mechanism, the hypothetical phlogiston, so that every process of the type we are discussing was supposed to involve the gain or the loss of this almost imponderable substance. By this mechanism they fell. They had not yet recognized that the air is a chemical reagent, and thought that the process of burning was merely the loss of phlogiston. When it was found that substances in burning gain in weight they were obliged to retreat before the proponents of the oxygen theory. If they had only thought to say "The substance burning gives up its phlogiston to, and then combines with, the oxygen of the air,"

<sup>1</sup> Lewis, *Valence and the Structure of Atoms and Molecules*.

the phlogiston theory would never have fallen into disrepute. Indeed, it is curious now to note that not only their new classification but even their mechanism was essentially correct. It is only in the last few years that we have realized that every process that we call reduction or oxidation is the gain or loss of an almost imponderable substance, which we do not call phlogiston but electrons.

The art of weighing brought into chemistry a quantitative, and therefore mathematical, element. The discovery of the law of multiple proportions led Dalton to his atomic theory. I shall not dwell upon the modern development of this idea in the hands of chemists and physicists, nor show how the atom has been brought closer and closer into our field of vision, so that we have been able to dissect it into its component parts. Many details of our atomic picture may still be erroneous, but concerning its cardinal features most scientists are now agreed.

It may, however, be instructive to compare the methods by which physicists and chemists have attacked this problem. The physicist starts with a small number of measurements, which, however, are of the highest accuracy. He finds an equation which will fit these measurements

with an accuracy of one part in ten million. Later he has the genius to discover that a simple model following laws closely akin to those of classical mechanics and electromagnetics would give the phenomena which have been observed. From his simple model he proceeds to more complex ones, involving a greater number of parts, and calculates as best he can the perturbations produced in part of his model by another part. And thus from his model as a center he works out into the unknown.

The method of the chemist is radically different. It is convergent rather than divergent. His data are far less exact, but they are vastly more numerous. Some are rough measurements, but the greater part are not even metrical in character. They are based upon the observations of thousands of different substances, and from these observations come rough generalizations like the periodic law of Mendeléef. And so he works gradually through the complex molecule into the very heart of the atom.

The history of science shows that we must not rank either method higher than the other in power or attainments. Yet those who follow the first of these two great methods of scientific inquiry are apt to look askance at the results



of the other method. After the beauty and precision of their own type of experiments and deductions they do not see the cogency of inferences drawn from such a vague chaos. Some years ago I discovered a universal tendency of electrons to pair with one another in the molecule and in the atom, and this conclusion is now pretty generally accepted by chemists, but it seems to conflict with certain equations for stability, and for this reason it has not been fully accepted, as far as I know, by any physicist. Yet not merely the chemical facts, but the observations by the physicists themselves upon ionizing potentials and spectroscopy, show nowhere so remarkable a distinction as that between substances which have an even and those which have an odd number of electrons in the outer shell of the atom.

It must be admitted that science has its castes. The man whose chief apparatus is the differential equation looks down upon one who uses a galvanometer, and he in turn upon those who putter about with sticky and smelly things in test tubes. But all of these, and most biologists too, join together in their contempt for the pariah who, not through a glass darkly, but with keen unaided vision, observes the massing



of a thundercloud on the horizon, the petal as it unfolds, or the swarming of a hive of bees. And yet sometimes I think that our laboratories are but little earthworks which men build about themselves, and whose puny tops too often conceal from view the Olympian heights; that we who work in these laboratories are but skilled artisans compared with the man who is able to observe, and to draw accurate deductions from the world about him.

In the snobbery of science each branch attempts to rise in the social scale by imitating the methods of the next higher science and by ignoring the methods and phenomena of the sciences beneath. Indeed, it is a common fault of mankind to refuse to recognize the existence of a phenomenon unless some mechanism has been devised or, as we say, some explanation is offered. Boswell,<sup>2</sup> speaking of Kenneth Macaulay's *History of St. Kilda*, says: "Macauley told me he was advised to leave out of his book

<sup>2</sup> Boswell, *Life of Johnson*. In this connection Mr. Christian remarked: "The situation of St. Kilda renders a north-east wind indispensably necessary before a stranger can land. The wind, not the stranger, occasions an epidemic cold." We have so great an instinct to seek out cause and effect that we often admit a phenomenon only after some explanation is offered, although the explanation may be a little more incredible than the phenomenon itself.

that wonderful story that upon the approach of a stranger all the inhabitants catch cold; but that it had been so well authenticated he determined to retain it."

Organic chemistry is one of the less mathematical sciences. The whole theory of structure requires about as much mathematics as a child needs for building houses with blocks, and while the balance has been a useful adjunct, exact analysis has served rather as a corroboratory than as a primary method of research in organic chemistry. The science has grown, like the biological sciences, through a dual method of classification. On the one hand, we study the actual properties of substances; on the other, their genetic relations. From a familiar substance a new one is made, from this another. Some of these products can be prepared from other initial substances, and so these threads of interlocking relationships have been woven through the whole complex material.

The atomic theory made it possible to prepare models, first of the simpler and later of the more complicated molecules. The rules of this kind of architecture are illustrated in Figure 25. A hydrogen atom (indicated by H) has one point of attachment, the oxygen atom (O)

has two, the nitrogen atom (N) usually has three, and the carbon atom (C) has four. Thus hydrogen can be attached to only one other atom, oxygen to two others, and so on. By means of various chemical reactions it is possible to make one atom or group take the place of another, and in most such substitutions it has

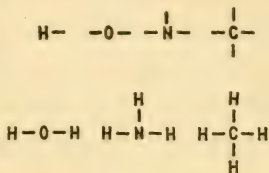


FIGURE 25  
Atomic Valences

proved justifiable to assume that the structure of the rest of the molecule is unaffected. In this way it is possible to build up or to synthesize complex molecules of known composition.

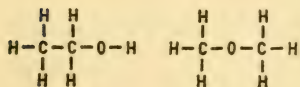


FIGURE 26  
Isomers

Sometimes two molecules, as shown in Figure 26, have the same atoms but in a different arrangement, and the two are known as

isomers. The number of possible isomers that can be prepared from a given set of atoms increases with enormous rapidity as the number of the atoms in the set increases. This number of isomers not only soon exceeds the total number of known substances, but a molecule with

only a hundred atoms would have so many possible arrangements that merely to give the formulæ alone would require more books than are contained in all the libraries of the world!

Many of the molecules produced in living cells contain thousands of atoms, and here we have an interesting case of a set of things which we know to be a discontinuum aping the manners of a continuum; for, as we pass from the simple to the complex structures, the difference between each substance and its nearest neighbor grows smaller; and the substitution of atoms and groups for one another, although each is a distinct act, makes such slight and apparently gradual changes in properties that the skilled organic chemist can mold and remold to his desire, as though he were working in a plastic continuum.

So, as the organic chemist acquires proficiency in this art, for indeed it is almost an art, he acquires an intimate *acquaintance* with his material. This leads him to a few great generalizations, to a large number of working rules of limited or sporadic applicability, and to many vague guesses or little tricks of thought, which he cannot or will not impart to others. In fact, much of his knowledge does not fully emerge



into his own scientific consciousness, and has been called chemical instinct (*chemisches Gefühl*); yet it is extraordinary with what precision he will calculate the properties of a substance he has never seen or the consequences of a reaction he has never tried. It is amazing to find how few of the structures which he has assigned to the various molecules are still in doubt.

A moment of hesitation came when the work of Pasteur required the chemist to explain the so-called optical isomers, which are alike in number and kind of atoms, and in almost every other physical and chemical property, but which rotate polarized light in opposite directions and by equal amounts. But LeBel and van't Hoff showed that the tetrahedral carbon atom, which had already been invented, not only could account for these substances, but should have predicted them. Figure 27 shows two atomic structures which are thus alike in the sense that a man is like his image in a mirror, or that a right- and left-handed glove are alike.

If four different atoms arrive fortuitously where a carbon atom is and combine with it, one arrangement is as likely as the other; never



shall we find a better illustration of a level chance. Therefore when a process forms billions of such molecules it would seem like an extraordinary fluctuation from the average if we should ever find a discernible excess of the right-hand or left-hand molecules. Never, in fact, has

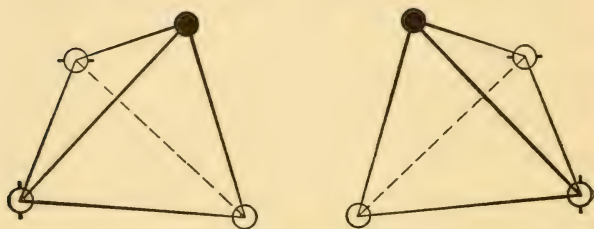


FIGURE 27

Right- and Left-handed Molecules

such an excess been found in reactions which start with substances which are not themselves predominantly right- or left-handed, and which are not subjected to some peculiarly asymmetrical environment. If the reaction produces right- and left-handed molecules it produces them in equal quantities. By crystallizing the product crystals are formed which are themselves right- or left-handed, but the two varieties occur in equal amounts.

By scrutinizing these crystals we may pick out, as Pasteur did, the one kind or the other, and thus obtain a right- or left-handed substance; but this is an exercise of *conscious choice* on our part. Now it is of the greatest interest to observe that not only we ourselves but other living things are capable of separating the right- and left-handed substances. While their separation in the inanimate world would be an extraordinary occurrence, in the world of animate nature it is the rule and not the exception. There is probably not a single living cell which does not contain thousands of optically active substances, and this is one of the most striking attributes of living things.

The whole complex field of organic chemistry is chiefly due to vital processes. Of course the old sharp distinction between inorganic chemistry and the chemistry of living tissues went the way of all such distinctions when Wöhler discovered that a typical organic substance could be synthesized in the laboratory. But after all Wöhler was a living being and merely produced outside of himself what other men produce inside themselves. If one could only know this world as it would have been without the intervention of men and microbes, he would not even

guess the existence of the great domain of organic chemistry.

One by one we have seen how categories, which at first seem sharply defined, merge one into another, and how every classification when analyzed shows that some imaginary line has been arbitrarily taken as a boundary. What shall we now say to this prime classification of science into its animate and inanimate branches? Do living beings possess traits which the rocks do not possess, or are the same traits possessed in different degree? Indeed, are all distinctions in kind reducible to distinctions in degree? These questions are hard to answer, but nevertheless I should like to discuss them a little.

I have already spoken of living things as cheats in the game of entropy. They alone seem able to breast the great stream of apparently irreversible processes. Those processes tear down, living things build up. While the rest of the world seems to move toward a dead level of uniformity, the living organism is evolving new substances and more and more intricate forms. Is it possible, however, that we may find some missing link to connect the animate with the inanimate?

Living creatures have been characterized by

their capacity for reproduction, but it has been pointed out that in a minor way crystals have the power of reproduction. If into this beaker (containing a supersaturated solution of sodium thiosulfate) I drop a minute crystal, it grows. I take a bit of this new growth and inoculate a second beaker, and so I may go on. Some years ago in our laboratory we wished to determine some properties of crystallized glycerine. We tried in every known way to make glycerine crystallize, but in vain. However, we discovered that some crystallized glycerine was in storage at a certain place in British Columbia, and importing a sample of this, the laboratory at once became "infected" by glycerine crystals, and no matter what new sample of glycerine was used there never was any difficulty in making it crystallize.

There is a peculiar phenomenon known as tin disease or the tin pest. In cold climates organ pipes, roofs, and other objects made of tin often disintegrate and fall into dust, and a sort of quarantine has to be established to prevent the disease spreading as an epidemic from those pieces of tin which have once become infected. Yet this all proves to be due to the fact that the gray, powdered tin is a more stable form at

low temperatures, and when it is brought into contact with block tin it spreads as the crystals just now spread in the beaker.

A salt often crystallizes in a number of different ways, but these ways seem to be fixed through the ages, and the crystals that we find in geological deposits are not distinguishable from those which we now grow in the laboratory. But the horse, while apparently much the same from one generation to another, is very different from his tiny five-toed ancestor, which in its turn probably bore little outward resemblance to the reptilian ancestor that was living at the time of the salt deposits of which I have just spoken. I must present later my apologies for violating one of the most sacred taboos of biological science when I state that the essential difference between the reproduction of the crystal and the reproduction of an organism is that the latter is reproduced with the transmission of acquired characteristics. Inanimate things we describe as obeying laws which are fixed for all time, but the living organism is an opportunist, making new laws from time to time in its constant evolution.

It is easy enough to make such an abstract classification, but do we know that any sub-



stances precisely conform to it? May there not be, as I suggested before, some missing link between living and non-living things? Having examined the reproduction of crystals we may next view another analogous phenomenon, which in most cases is still mysterious. It has long been known that chemical processes are sometimes enormously accelerated by minute traces of foreign substances. Such remarkable action is named catalysis, but this, as Poincaré would say, "is not to solve the difficulty but only to baptize it." Nevertheless, giving a name to a phenomenon does indicate that we have recognized its existence and its generality, and when we state a problem clearly we are already part way toward its solution.

Now one of the most interesting kinds of catalysis is the one in which a reaction is accelerated by one of its own products, so that a long time may elapse before anything happens, but if that product begins to form, or is introduced from without, the reaction goes faster and faster. This autocatalysis has not long been known, but we already realize that it is of extremely frequent occurrence, and doubtless we shall know much more about it in the near

future. In the meantime we must deal with cases which are somewhat hypothetical.

Let us imagine that a certain solution is capable of producing a given organic substance, but that it will not produce it unless one molecule of this substance is already there, after which more and more of these molecules form at the expense of the nutrient solution. We are assuming that all of these new molecules are the same in composition and structure as those first introduced. Now we may assume further that an isomer of this substance, that is, a substance of the same atoms in a different arrangement, would also reproduce itself in the same solution. Thus by starting with one or the other of the two isomers we could propagate it by inoculating new quantities of the nutrient solution with portions of that which had already been inoculated. Or, if both were growing in the same solution, we could form cultures and by careful manipulation separate the one type from the other and afterwards breed them true.

Now the molecules of such isomers occasionally go over into one another, perhaps some collision with a neighboring molecule knocks off an atom or group which falls back into a new position. If then we started with one of these

substances we might, after a certain number of cultures, suddenly find the other appearing as a mutation, and this would then breed true. If there were only two isomers we should find occasional changes back and forth from one to the other. But suppose that it is a complex molecule with a still finite but practically countless number of isomers, then the chance would be negligible that any such mutations would go back to a form that had previously existed. We should see a process of evolution, each molecule reproducing itself exactly, until an accidental rearrangement would set a new molecule to propagating itself. Would not this be reproduction with the transmission of acquired characteristics?

You may object to my using terms drawn from biology; but suppose that this hypothetical experiment could be realized, which seems not unlikely, and suppose we could discover a whole chain of phenomena, leading by imperceptible gradations from the simplest chemical molecule to the most highly developed organism. Would we then say that my preparation of this volume is only a chemical reaction, or, conversely, that a crystal is thinking about the concepts of science? Nothing could be more absurd,

and I once more express the hope that in attacking the infallibility of categories I have not seemed to intimate that they are the less to be respected because they are not absolute. The interaction between two bodies is treated by the methods of mechanics; the interaction of a billion such bodies must be treated by the statistical methods of thermodynamics. They are the same bodies and presumably follow the same behavior, but a great group of new phenomena *emerges* when we study an immense number, and by this we must mean merely that phenomena appear that never would have been recognized or dreamed of if the two bodies alone had been studied.

Now let me make my apology for venturing to say that one of the most striking attributes of living organisms is the transmission of acquired characteristics. While the statement can hardly be questioned if there is no implication as to the mode or mechanism of acquiring the characteristics in question, yet I have used a phrase which has long been the catchword in one of the most bitter of biological controversies. An animal during its life is constantly changing in consequence of its habits and environment; for example, certain of its muscles



become strengthened through repeated use. In general it may be said to be always adapting itself to the circumstances under which it lives. At one time it was generally supposed that such adaptation of the individual could in large measure be communicated to its offspring, thus providing a means for the adaptation of a whole species to changing conditions, more rapid than natural selection, and independent thereof.

This is the theory that has been repudiated by modern biology. Indeed, the evidence which has so far been accumulated shows that there cannot be much inheritance of physical traits which have been acquired in the lifetime of an individual. It is true there are certain curious facts still to be explained. In examining the fossil remains of man, it is seen that in the earliest stone age, when food was consumed in its natural state, the human tooth was very large and powerful, and at each successive period the tooth has kept well adapted to the current habits of eating. The changes appear to have been more rapid than could be readily accounted for by the survival of the fittest, since unnecessarily strong teeth could hardly be a serious handicap; but there are many unknown



factors here, and a thoroughgoing follower of Weismann might even claim that, conversely, the invention of the flour mill and the oven had followed upon the diminishing strength of the tooth, necessity being the mother of invention.

On the whole the disbelievers in transmission of acquired physical characteristics, in the narrow sense in which we are now using the term, seem to have much the better of the argument. They are, moreover, supported not only by experiment and observation but also by the whole development of biological theory.

In the beehive with its thousands of individuals only one member (except in the rare case when a worker lays eggs) becomes the mother of the next generation. So it appears in plants and animals there are myriads of cells which do the work of the body, but only a few, the germ cells, that serve to perpetuate the species. Unless therefore there is a higher degree of intercommunication between the two types of cells than is commonly supposed, we should hardly expect anything that happens to the worker cell to have any marked effect upon inheritance. This idea of the independence of the "germ-plasm" is entirely in accord with the recent developments in biology to which the redis-

covery of Mendel's law and the study of chromosomes have given an impetus comparable to that given to mechanics by the discoveries of Galileo and Newton.

It seems that every individual organism, as well as every typical cell, has a dual nature. As the animals entered the ark two by two, so the various factors of heredity enter the cell in pairs. On the other hand, the special cells of reproduction result from a process of bisection, so that each possesses but one member of every pair. When such paternal and maternal cells unite, the new individual once more possesses the complete quota of pairs, but it has only half of the heredity factors of the father and only half of those of the mother. It appears to be an absolutely level chance which half of any one paternal or maternal pair enters into the offspring. This is a rough statement of the famous law of Mendel, which reduces the whole science of genetics to a simple problem in the theory of probability.

As so often happens in science, this law of Mendel was almost forgotten until a mechanism was discovered whereby it could be interpreted. The modern investigations of chromosomes have brought into biology a degree of unity

comparable with that given to chemistry by the study of the atoms and the sub-atoms. As the atoms are found to be the same in the stars and on the earth, so the geneticists have discovered the same mechanism of inheritance in all the higher organisms. The chromosomes of plants and animals are so much alike that when seen under the microscope a trained observer would frequently be at a loss to say which was which.

Mendel's law has not only given a remarkable incentive to scientific research, but it is beginning to exert a wide and powerful influence upon the practical breeding of plants and animals. Yet when we inquire whether this law is true, or, in other words, whether it is precisely applicable to all cases which are assumed to come within its scope, the answer will probably be in the negative. The law states that the traits of any one generation are determined according to purely statistical laws by the traits of the ancestors, and by nothing else. Tower's<sup>3</sup> observation, of an apparently new species obtained from the potato beetle by breeding in a refrigerator, has not yet been successfully duplicated. Yet there seems to be no question that environment plays some part in determining the trend

<sup>3</sup> *Carnegie Institution Publications*, No. 48 (1906).

of heredity. Even such a phenomenon might possibly be reconciled with Mendel's law when we consider that *natural selection* must be operating even among the reproductive cells themselves, of which "many are called but few are chosen." Yet it seems likely that further investigation will show some departure from the general law that chance alone determines heredity.

Yet if the wish of biology is to resemble physics and chemistry, that wish is fulfilled, for we have seen how each law of the physical sciences has in course of time been supplanted by a more general or a more accurate law—a process which we may expect to see continued as long as science itself continues. So Mendel's law, and the body of biological knowledge which has followed in its train, represent a great first approximation to the understanding of heredity, which, in its turn, will be followed by second and third approximations as we travel onward toward the receding horizon of scientific truth.





## VIII

### *Life; Body and Mind*

NOW I come to my most difficult subject, and one which perhaps I should not be discussing at all; but in the general scrutiny of the structure of science we cannot omit the most interesting part of all. Besides, we have seen how subtle the distinction may be between inanimate and animate things, and perhaps some consideration of biological facts will enlarge our view of lifeless things, even as some excursion into non-Euclidean geometry enhances our appreciation of the Euclidean. It may even be amusing to the biologist to see how some of his problems look to the innocent bystander. If I say some things that seem uncouth according to his fashions it will not be for the sake of provoking resentment, and I shall hope from him a good-natured tolerance rather than a feeling of irritation.

Indeed, it is easier for an outsider to say some things which the biologist himself might occasionally wish to say, for he is in an awkward situation. Of all the natural sciences, his touches most closely those human passions and preju-

dices which disturb the calm contemplation of nature; his also is the subject which everyone feels competent to talk about—even a chemist.

It is not therefore surprising that he has been willing to accept and to impose upon his fellows a sterner discipline and a greater measure of prohibitory legislation than is elsewhere the custom. If I ignore some of these prohibitions I shall probably be called names; perhaps already some who have read my last two chapters will have called me a vitalist, but I confess to so complete an ignorance as to the meaning of the term that I should not know whether or not to be pleased by the epithet.

Certainly there is no necessity for invoking a "vital force"; for after all it is a crude sort of science which attributes all physical processes to mechanical and electrical forces; all chemical processes to chemical forces; all vital processes to vital forces. But everyone must admit the existence of great phenomena lying far beyond the petty domain which is under the nominal jurisdiction of the laws of physics and chemistry. If I am forced to acknowledge some creed with respect to vital phenomena, I shall be like the backwoods farmer who was asked if he be-

lieved in baptism. "Believe in baptism!" he said, "I have seen it done."

It is recognition of the peculiar phenomena of life which has led to the classification of biology as an independent science. We may doubt whether there is an impassable gulf between vital processes and those of physics and chemistry; but need I point out again how indispensable to the growth of science are its classifications, although its several categories rarely, if ever, maintain their integrity after prolonged analysis. By some such means as I sketched in the last chapter it may be possible to pass by insensible gradations from the typical system of the biologist to the typical system of the physical chemist. We have a longing for unity, and since nowadays we have excluded the supernatural by making the word nature comprehend every kind of occurrence, so we might define science as the search for relationships between all sorts of existing phenomena. Then there would be only one science, but we should not attempt to identify it with any one of its present subdivisions.

It was once supposed that sun, clouds, winds, mountains and brooks were each endowed with psychic purpose, and that every occurrence in

nature was an exhibition of such purpose. Then by way of reaction came the sway of mechanism, which bravely strove to embrace every process, from the revolution of the planets to the writing of Hamlet; and the lineal descendant of this mechanistic philosophy is the physicochemism of the present day. Certainly far more may be accomplished by studying the physical chemistry of vital processes than by speculating about a thunderstorm's purpose or an atom's loves and hates, for there is no evidence for them at all, while many processes in living cells are identical with those that we meet in laboratories of physics and chemistry; and there are many others which will be interpretable in a larger physical chemistry which the further study of these vital processes will itself bring into being.

Nevertheless, it is indisputable that many of the characteristic properties of living beings are not only far beyond the reach of existing physical science, but are not even suggested by the most remote extrapolation of the laws and theories that we have made to fit the inorganic world. We see no limit to the interesting and useful results that will inevitably come from a further application of the methods of physics and chemistry to the physiology of animals and



plants. Yet the belief that even an infinite succession of such investigations would ultimately lead to a comprehensive understanding of vital phenomena seems to be one of those illusions, like the *ignis fatuus* of the mechanistic philosophers, which blind our eyes to many interesting trails that should tempt the scientific explorer.

Perhaps our genius for unity will some time produce a science so broad as to include the behavior of a group of electrons and the behavior of a university faculty, but such a possibility seems now so remote that I for one would hesitate to guess whether this wonderful science would be more like a mechanics or like a psychology. Indeed, if all the sciences now existing could be brought within one great scheme, would not this very process of unification be accompanied by the discovery of great bodies of new and equally mysterious phenomena, the existence of which has not yet been suspected?

The whole idea of evolution, which during the last half century has penetrated so deeply into every branch of philosophic thought, seems to be essentially connected with the one-way time which we discussed in a previous chapter. Evolution implies constant growth, leading to new combinations, new substances, new forms and



new behavior. In a century we have seen electricity grow from a mere toy into one of the most powerful instruments of civilization; and because we have followed this development through its successive stages we experience only a mild sense of mystery.

To inquire into the origin of life is like seeking the origin of electrical machinery, or the origin of music. Every increase in complexity of arrangement, of form, of substance, leads to new and often incalculable properties. A cabinetmaker who has never seen a violin may recognize the wood of which it is composed, or admire the nicety of construction; but he cannot guess of the melodies which will be drawn from this instrument by the hand of a master. If some accident breaks the violin to bits it still has the same atoms and molecules attracting and repelling each other in the old way, but the tone of the violin is forever lost. Every complex structure is more than the sum of its component parts, and if a wireless set is demolished, or a living creature dies, something was that is no more.

Sometime in the latter geological history of the world an observer from another planet might have seen a scum spreading over the

earth's surface, and he might speculate, as we now speculate, in vain, asking whether this was a gradual development of processes which he had formerly failed to recognize, or whether the germ of some new thing had drifted in from extraterrestrial space. We see no way of answering this or the related question as to whether life was born only once upon the earth or has begun many times and may even now be beginning again. The identity of the mechanism of reproduction in plants and animals, which I mentioned in the last chapter, strongly indicates that at least the higher forms of life had a single origin. The dodo once extinct will never be seen again, and if all existing life should become extinct we have no way of guessing whether this particular kind of complexity in nature would ever reappear.

Shall I now be called a vitalist if I say that many of the phenomena of life may be studied by methods which, while truly scientific, are entirely independent of the methods of geometry, mechanics and chemistry? Of these phenomena, which have no counterpart, as far as we know, in inanimate nature, one of the most striking is known as the struggle for existence, an idea

which was first clearly advanced by Malthus<sup>1</sup> in that essay which not only profoundly influenced the whole development of political science, but was an acknowledged inspiration of the work of both Darwin and Wallace. Often as I walk over the sand dunes by the Pacific I am amazed by the incessant warfare between the agents of destruction—the tides, the wind, and the drifting sands—and the constructive principle of vegetable life. Trees, maimed and dwarfed, still wage their battle against the bitter winds; whole gardens spring up only to be washed away by a single wave; yet if only a root or a seed remain, once more the work of construction begins. We never feel sorry for the atoms and the molecules, and it must be some feeling of kinship that makes us see pathos in this apparently purposeless waste. But, tragic or not, the struggle goes on and will go on wherever living creatures exist. By constant

<sup>1</sup> "Plants and animals . . . are all impelled by a powerful instinct to the increase of their species; and this instinct is interrupted by no doubts about providing for the offspring. Whenever, therefore, there is liberty, the power of increase is exerted; and the superabundant effects are repressed afterwards by want of room and nourishment, which is common to plants and animals; and among animals, by their becoming the prey of each other." Malthus, *An Essay on the Principle of Population*.

*trial* the organism finds ways to circumvent the agencies of destruction. The fish lays millions of eggs that a few may survive, and not only individuals but species die. Very few of our existing species will be the ancestors of those that will exist a few million years hence.

When we describe this *struggle* for existence as a phenomenon apparently independent of, and even contrary to, the generalizations of inorganic science, we are using, intentionally or unintentionally, a word which suggests something purposive or mental, and, indeed, there is a great group of vital phenomena which can hardly be discussed without using some words which have been used, in a narrower sense, to describe the habits of mankind. If, for example, we use the word *mental* when we speak of other living things, it is evident that we must enlarge or extend its primitive meaning. But this is only doing, in a less exact way to be sure, what we have already done with the concepts of number and distance, in the first two chapters.

Our minds work through analogies, and when we attempt to formulate the behavior of living things we cannot forget that there are two kinds of behavior with which we are already intimately acquainted: on the one hand, the be-



havior of weights and electric charges and chemical reagents; on the other hand, the behavior of man. These require two distinct vocabularies, and most writers who describe animal behavior have adopted the one or the other. We have the "nature fakers," who make animals think and act just like men, and there are the others, who regard the swarming of bees as a sort of chemical reaction. I do not know which of these two extremes to regard as the more futile, for both extrapolations go far beyond what is now justifiable. Yet the attempt to bridge this vast gulf is a legitimate aim of science.

Extremely interesting results have been obtained through a careful study of those blind tropisms which drive organisms hither and thither without any apparent choice of their own. We see creatures impelled by gradients of temperature or concentration, by light and by chemicals, and we suspect that this willy-nilly obedience to such tropisms occurs also in more highly developed creatures, and may explain many of the acts of man. But there is a prevailing taboo against any attempt to trace in the opposite direction the mental processes of man into the animal kingdom. Even the mind of man



has recently fallen under the ban of science, and is studiously disregarded or treated as a trivial by-product of chemistry and physiology. It is hard to understand this fashion unless it is that, distrusting ourselves, we are afraid of once more anthropomorphizing the world and repeopling it with its old sprites and goblins. I believe we may resist this temptation while we devote the rest of this chapter to a discussion of what in an extended sense we may call the mental phenomena of animate things.

It is surely some kind of make-believe that has led scientists in recent years to treat of the mind as some sort of minor parasite upon the body. A physical injury produces a change in our mental state, but on the other hand it may be shown that an insult or a disappointment immediately affects the chemical composition of the blood. If we look for cause and effect, or for temporal precedence, we must regard the physical and the psychical as of coördinate importance in the life of man. Surely it is not really necessary to argue for the power of mind? Consider the progress of science and invention. If all these were blind consequences of chance arrangements of atoms, why should not the age of alchemy follow rather than precede the age

of chemistry? Why should not the efficient automobile of to-day be replaced by the horseless carriage of to-morrow? Among the animals too it would be a grave mistake to ignore the mental traits. Darwin, while advancing the theory of natural selection, also pointed out the great importance of sexual selection; but that must certainly be regarded as a mental selection by everyone except the most extreme psychophobe.

If, however, we say that a living individual has both body and mind, and that each has a profound influence upon the other, we must be careful not to say more than we mean. Indeed, I may remark parenthetically that it is not always easy to recognize the individual<sup>2</sup> itself, to say nothing of the individual's body and mind.

Ordinarily, however, there is no ambiguity when we speak of an individual, but can we so easily segregate the body from the mind? We

<sup>2</sup> Maeterlinck, in his beautiful book, *The Life of the Bee*, speaks of the spirit of the hive, and although scientists might not be willing to go quite so far as this at present, it may not always be easy to tell when a colony of individuals is itself to be regarded as an individual. Professor G. H. Parker is making some extremely interesting studies of a type of colony which to certain stimuli behaves like a single creature, to others like an assemblage of mutually independent members.

recognize in an organism certain phenomena which we call material, and others which we call mental, and therefore as a sort of shorthand we talk of body and mind. But when we say "the body has a mind" or "the mind has a body," we may be falling into the same sort of confusion that I discussed in the third chapter, where we saw that the physicist, in trying to understand the phenomenon of light, obscured the problem by inventing an unnecessary ether, to which he ascribed properties additional to those of light itself. If in order to avoid the trouble of saying that the living individual exhibits both material and mental phenomena we simply say it has body and mind, let us imply no more than when we say that an electron has a charge and a field, recognizing that they are but different aspects of the same individual.

When we come to examine the mental traits of man we may roughly divide them into two groups, one of which refers to the degree of adaptation to the existing environment, and the other refers to adaptiveness to a changing environment. The first class suggests some such term as *wisdom*, the second may be classed as *intelligence*. The first includes traits that show some profiting by experience, such as memories,

acquired habits, instinct, knowledge, skill; the second includes ability to meet new situations, to learn something from each encounter with a new problem, and thus to accumulate knowledge and skill.

We find it nearly impossible to interpret the behavior of animals except by ascribing to them mental traits quite analogous to these human traits, and we employ the same words with the understanding that they are now being used in a somewhat generalized or extended sense. When we compare animals with man we are tempted to assume naïvely that we can mark the human mind one hundred per cent in each of the several mental traits, and then rate the animal mind by finding how nearly it approaches the human standard in each particular. Thus we consider the animals most intelligent whose behavior is most nearly human. I have just been reading two books (admirably exemplifying a new and most hopeful movement in psychology),<sup>3</sup> which show the extraordinary aptitude of the anthropoid ape in the use of tools, ladders, and other external means for securing its ends. We are tempted to say that these apes, next to man, are the most intelligent of animals.

<sup>3</sup> Köhler, *The Mentality of Apes*; Yerkes, *Almost Human*.



But we must remember that the use of tools is only one way of showing intelligence, and one that is preëminently human. I doubt very much whether the ape could ever be as successful in herding sheep as the dog is.

Indeed, it is quite impossible to place man at the head of the list with respect to each of these mental traits. With respect to one—instinct—he occupies a quite inferior place. The rudimentary character of instinct in man seems to be due to his having found a substitute. The use of tools is but one symptom of what might be called externalization. The bee stores honey for posterity, the bird teaches its young to fly, but what are these compared with the enormous hoards of material and spiritual wealth that man accumulates from generation to generation! His habits of instruction, his stores of oral and written lore, his schools, his universities, his libraries—all these external accumulations represent the greater part of that adaptation to environment which in the human species has nearly taken the place of instinct.

The enormous power which this habit of hoarding has given him, perhaps leads man to overestimate his other talents. If we should define intelligence as ability to meet successfully



situations of an entirely new character, it is generally conceded that modern man is no more intelligent than his Cro-Magnon ancestors, whose total material wealth was represented by a few beads and implements of stone and ivory, and whose lore fell equally short of the accumulated knowledge of to-day.

It is indeed doubtful whether, judged by any objective criterion, the intelligence of man is supreme. It is hard to find such a criterion which is fair to both parties. We are apt to introduce into our test some element in which the one species or the other shows an expertness that comes from habitude. Thus man certainly has exceptional practice in the art of communicating his ideas; yet if we consider intercommunication between men and animals we find that we have little, if any, advantage. Of the various desires that I might wish to communicate to my dog he can understand about half a dozen, and can make me understand about the same number of his own.

Our present era, which is sometimes called the age of science and invention, might perhaps be more appropriately designated as the age of publicity and conservation. In former days, inventions and discoveries often died with their

discoverers, but now there are fewer of these "mute inglorious Miltons." Every discovery, every trifling invention, every new trick of salesmanship is at once broadcast from any part of the world and garnered into our large granaries of knowledge.

If now we cease to make invidious distinctions between man and the animals, and consider the latter by themselves, it seems natural to class together the three traits that we call memory, habit and instinct. They relate to phenomena of quite similar aspect, and while we say that memory is acquired during the life of an individual, and that instinct is acquired over a much longer period by the species, still it may be that the distinction between an individual and his ancestors is far less sharp among the animals than it is with man. When we consider the continuity between the more complex forms of life and the simpler unicellular organisms which propagate by simple fission, where it would be absurd to call one half the parent and the other half the child, it seems unwise to insist upon too sharp a distinction between an individual and his ancestors.

While, therefore, we might be inclined to treat instinct as merely inherited memory, we

cannot logically do so as long as we adhere to the dogma of the non-inheritance of acquired characteristics. I have already said something of the evidence against the transmission of physical characteristics acquired through use, and passed on in such manner as to increase the adaptation of a species to its environment. It seems to me, however, that in the case of acquired *mental* traits and habits, the evidence against transmission is by no means so strong. Here also orthodox biology insists upon its formula of non-transmission, but now other views are at least being considered by numerous biologists.<sup>4</sup>

Some years ago as I passed a bookstall in France my eye was caught by the title *The Psychic Life of Insects*.<sup>5</sup> I found not only that the book had been written by one of the soundest and most experienced of French naturalists, but was also delightful reading. After a careful and sympathetic review of various investigations of tropisms, the author proceeded to vital

<sup>4</sup> Among the recent treatises on biology the one with which I have found myself particularly sympathetic is the very readable book by Professor J. Arthur Thomson, *A System of Animate Nature*.

<sup>5</sup> This fascinating book by Bouvier has since been translated into English by L. O. Howard.

rhythms, such as the seasonal or daily changes in plants and animals. A creature that has long been responding to daily changes of light and darkness will continue to show these daily variations for some time after it is placed in a room of constant illumination; but if subjected to a new period of alternation, a new adjustment finally comes whereby the creature has adapted himself to his new circumstances. It is such adaptations that show how far even the simplest organisms are removed from mere mechanisms. Then the book goes on to deal with the instincts which are so extremely characteristic of the insect world—instincts so nearly immutable that they persist through geological epochs, and yet not altogether immutable. The insect often shows its power to invent or learn new methods; and when we see the mass of evidence that Bouvier has brought forward it is hard to doubt his conclusion that this learning is in some measure transmitted to posterity.

For example, an insect accustomed to eating the leaves of a certain plant is forced with great reluctance to eat the leaves of a different plant; but after two or three generations the insect not only prefers the new food, but even, having an option, lays its eggs upon the new plant. This



may, however, be due to a tendency of the adult insect to seek the plant to which it has become accustomed in the larval state.

There are a great many minor observations which seem to indicate the transmission of mental characteristics acquired in a very few generations, although it is possible that each one would be ruled out by a very rigorous exclusion of evidence. Darwin believed the story of travelers, that birds in newly discovered islands acquire a fear of man which persists when the islands are once more visited after a number of bird generations. But this idea has since been seriously disputed.

It is generally believed, by those who raise animals, that a wild strain, such as a strain of wolf in dogs, leads to a number of generations of relatively untamable animals; but no one has made a really thorough investigation of this belief. I have myself noticed how easily a young dog whose ancestors have all been trained to hunt birds is excited by anything with feathers, and relatively uninterested in other forms of wild life; but this may be due to the selection and utilization of a trait already existing in certain canine varieties.

We know many kinds of creatures who have



made themselves at home in human habitations, like the chimney swallows; yet chimneys are a recent invention. You must all have noticed that in the early days of automobiles, when they were but few and traveled at low speeds, there was relatively far more destruction of hens and other domestic animals by automobiles than occurs now. These cases may perhaps be explained, however, by some sort of mutual instruction.

Yet the number of such observations, although no single one would be worthy of much notice, has led a number of investigators in recent years to plan definite experiments to test the transmission of newly acquired mental habits. Pavloff made a preliminary statement some years ago regarding some experiments of great interest, in which successive generations of mice, each subjected to the same training process, showed a remarkable increase in the rate of learning in the later generations. But he does not say how he excluded the possibility that the animals were being educated by one another. Experiments of this sort, which have in view the overthrow of so firmly established a dogma as the non-transmission of acquired characteristics, must be impeccable. Many years

ago I suggested an experiment to test the inheritance of acquired fear, using incubator chickens to avoid any possibility of instruction, and asked one of our leading geneticists what he would say if a positive result were obtained. He replied, "I would say that the experiments had been badly performed." I then asked him what he would say if he had himself performed the experiment. He answered, "I would not believe my own eyes."

Let us now turn our attention to another phenomenon which I have chosen, not merely because it is curious, but because its discovery is one of those crucial observations which has the power to change our whole attitude toward nature. Various authors have mentioned the numerous cases of apparent imitation of one species by another, but there is one case which is far more significant than all of the others that have been observed. If I should ask you what animal it is that has learned self-sacrifice and loyalty, that has developed a great industrial system with special laboring and military classes, that has built great structures fifty or a hundred times his own height, that has domesticated animals, and created an agriculture, with species of plants not to be found in the

wild state, you would probably say it is man; but man has been doing some of these things only a very few thousand years, while these same arts have been practiced for a million years by a lowly insect whose nearest relatives are the book louse and the cockroach. I refer to the termites, which are sometimes called white ants because of their remarkable resemblance in habits to the ant family. Yet the termites and the ants come at the very opposite ends of the classification of insects.

The resemblance is so extraordinary that Professor Wheeler<sup>6</sup> writes, "it is as if we had found, when Australia was first explored, the kangaroos and opossums enjoying a social organization like that of man." To ignore such a remarkable phenomenon is unscientific, to call it a coincidence is antiscientific. To apply repeatedly and against all principles of chance the hypothesis of coincidence is the very negation of science. If we had observed merely that the termites have a social organization, and that the group of insects to which ants, bees and wasps belong also have a social organization, it would be interesting. If it were observed that the termites and the ants are the only insects

<sup>6</sup> W. M. Wheeler, *Social Life among the Insects*.

which cut off their wings after mating, this might be a coincidence. But when we find that termites and ants have a cultural life which is almost identical; that each has its royal, worker, and soldier castes; that they have similar complicated methods of deriving sustenance from one another; similar building habits, with elaborately ventilated gardens in which are grown and cultivated fungi unknown in the wild state, and similar domesticated insects, how can we harbor even the suggestion of coincidence?

The parallel cited by Professor Wheeler is not quite exact. Supposing that, when Australia was discovered, we had found men and kangaroos building mud structures of the same architectural type, burying their dead in similar mounds and with like ceremonials, would we doubt that one of these species had exerted a profound cultural influence upon the other? Now ants and termites often live together in a happy symbiosis, and, indeed, there are certain species of ants which have been found only in the termitaria. These facts give us a feeling akin to that experienced by Robinson Crusoe when he found the footprints on his desert island. I do not claim that imitation is the only possible explanation of this curious phenome-



non, but I shall be surprised if any other can be presented which will do less violence to our established ways of thought.

Now finally it may seem absurd for me to venture, in my remaining page or two, even to touch upon the great problem of mind that has vexed so many generations of men—the classical problem of determinism or free will; but the solution that I have to offer I have already suggested in a previous chapter, and now need only restate it. The problem is not whether the mind is powerful—we have already discussed that—but is the mind free?

If we claim that the problem of freedom may be solved by the same methods whereby other paradoxes of science have been resolved, it is from no feeling of arrogance. The scientific puzzles of antiquity, the antinomies of later philosophers, have one by one been swept away before the advance of science. Why should this one problem be an exception?

Some of my colleagues have doubted whether this can be regarded as a scientific question, but it seems to me that nothing could be more so. No one science alone, but several sciences, physics, chemistry, geology and astronomy, assume as their very first postulate that the future of



any isolated system is completely determined by its past. Among all these sciences we know no single fact which contradicts this rule.

Nevertheless, as Johnson said to Boswell, "Sir, we know the will is free and there's an end on't." Indeed, a man may spend an evening expounding the theory that the will itself is the slave of purposeless laws of mechanics, but as he rises from bed the next morning, after a heroic effort of will, he cries, "and yet it is free." It is a paradox which comes almost daily to our attention, and never more than at the present time when the doctrine of determinism is very widely taught, and yet we have an uncommonly keen sense of individual responsibility, which is one of the main ideas underlying our whole social structure.

If I have at all succeeded in my attempt to show you science in perspective, and to illustrate how far we still are from any ultimates, many of you will already have guessed the mode of escape that I propose from this dilemma of freedom and determinism. I agree entirely with Johnson. There is no basic fact of any of the exact sciences which is so abundantly proved by the evidence of our observations as this freedom of the will. Presumably it would never have

been thrown into doubt but for a conflict with one of those absolutes to which we have clung, and still cling, so tenaciously.

Determinism is a great principle. It fits every known fact of inorganic nature; but like arithmetic or geometry we need not ask if it is true, but only how far it is applicable to the world of nature. In a previous chapter I suggested that in the game of chance played by the atoms and the molecules, even the simplest organisms are playing with loaded dice. The element of choice, which we allowed to living organisms, represents a departure—which may also exist in other cases, though not yet detected—from the simplest laws, of which the first is that the past determines the future. What we there called the element of choice we may now call free will.

He would indeed be bold who would attempt to estimate the degree of that departure from determinism which we call free will. The curvature of the earth is too small even to be detected by a person whose measurements are confined to the surface of a small lake, and the departure from our flat geometry of space-time which is necessary to account for the whole phenomenon of gravitation amounts to but one part in a thou-

sand million at the earth's surface. The errors caused by the assumption of the postulate of determinism may also be relatively small, but to ignore them and the phenomenon of free will would be like ignoring the existence of gravitation. The prisoner cannot escape from his cell through will power alone, and while the chemist may juggle his protons and electrons to make new compounds and perhaps new elements, yet he sees no way at present of increasing or diminishing their number. So on the mental and moral side we are limited to an extent that we do not always realize. Still these limitations of freedom need not blind us to the existence of that power of choice that characterizes living creatures, and has only been discovered in animate nature.

Let us not be confused by the double sense in which we use words like "physics" and "biology." We may well suspect that the subject matter of physics and the subject matter of biology constitute a single continuum, but the *sciences* of physics and biology comprise sets of man-made postulates and laws which no more need to be compatible with one another than do the geometries of Euclid and Lobachevski. The science of physics rests upon the postulate of

determinism; the science of biology, unless it is to ignore deliberately the phenomenon of behavior, must abandon this postulate and substitute therefor a postulate of choice or freedom.

We are justly proud of the likeness that we have achieved in this portrait of nature that we call science; but there is far more to be seen than we have yet seen, and nature itself is changing and growing. As in the laboratory, so probably also in nature, chemical molecules are daily manufactured that were never made before. As we continue the great adventure of scientific exploration our models must often be recast. New laws and postulates will be required, while those that we already have must be broadened, extended and generalized in ways that we are now hardly able to surmise.



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